2025年教学科研并重型副教授

职称评审支撑材料

（任现职以来的代表性成果 7 项）

申请人：季慧慧

学科：数学

院系：数学学院

目 录

1. Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems (中科院 1 区 TOP 期刊、特别奖励期刊) (1/3)

IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, VOL. 54, NO. 1, JANUARY 2024 97

Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems

Huihui Ji, Shengyuan Xu, *Senior Member, IEEE*, and He Zhang

***Abstract*—This article addresses the problem of stability of unknown nonlinear systems with a stochastic scheduling scheme. In order to solve the difficulty resulted by the unknown nonlinearity, the neural-network approximation technique is introduced. Notice that for the controller design of uncertain nonlinear systems, numerous simulation studies and actual industrial implementations show that the neural network is a good candidate to handle the design difficulty resulted by unknown nonlinearities. The feedback control signal in this article is produced based on periodic sampled data. In the stochastic scheduling scheme, both the choices of controllers and their execution time allotted to the scheduler are random. Sufficient conditions based on the probability distribution of almost sure stability are obtained by using a general Lyapunov functional and some stochastic techniques.**

***Index Terms*—Almost sure stability, neural network, stochastic scheduling, unknown nonlinear systems (UNSs).**

I. Introduction

IN EMBEDDED systems on the same platform, a general trend can be observed to implement all kinds of concurrent real-time (RT) tasks, thereby reducing whole hardware costs and development time. In these tasks, the implementation of the control algorithm has to be highly time critical and has usually taken a very conservative scheduling approach that statically allocates execution time. A result is produced that the overall architecture is often poor performance because it is very rigid, difficult to reconfigure for component additions or changes.

In the current automotive field, embedded electronic control unit (ECU) usually needs a multitask RT operating system, which dynamically schedules its tasks to meet to requirements both quality of service (QoS) and different load conditions. RT preemption algorithms, such as rate monotonic (RM) and the earliest deadline first (EDF), can suspend the execution of a task when there are other higher-priority task requests. In a hard RT system, some conservative assumptions, such as the estimate of worst-case execution times (WCETs) for tasks, are presented to keep the schedulability of given tasks and avoid missing deadlines. These assumptions will lead to inadequate development of computing platforms, thus reducing cost efficiency. Conversely, in industrial practice, where a given budget for computing power is required and fixed, the control algorithm must be greatly simplified so that it can be calculated within a specified time. This will degrade the whole performance of the controlled system. As a current trend, the design of embedded systems is to relax constraints of schedulability and to introduce computation models (i.e., "weakly hard" [2] and "resource reservations" [1]). Both branching depended-data-driven and using pipelines result the different execution times of tasks. Here, while the execution time of tasks would vary greatly, occasional deadlines can be tolerated. Difference from using the total WCET boundaries, a method based on QoS metrics is used to quantify the average miss value and the random distribution pattern, in which approach of design constraints according to QoS metrics are typical [3].

A. Motivation

Under the environment of soft RT, a strategy for designing and scheduling linear controllers is presented in [4] to control a discrete-time random jump linear system (RJLS). It not only overcomes the limitation of existing practice but also allows for better development with the same resources. Instead of analyzing RJLSs, Wang and Sun [5] considered a class of continuous-time RJLSs. The problem of exponential stabilization of RJLSs in almost sure sense is studied, where the systems are controlled by linear controllers via a random scheduling scheme. By applying a limit idea to the random transfer matrix of RJLSs, sufficient stability conditions of closed-loop RJLSs are obtained and those conditions are in the forms of linear matrix inequalities to easily solve the controller gain parameters. Especially, considering the dwell time of random jump mode and the probability distribution characteristics of random scheduling, it is verified that they taken positive roles in system performance. However, it is worth mentioning that the above results are obtained by analyzing the explicit solution of the discrete- and continuous-time linear systems. Those analysis methods are hardly to applied into nonlinear or hybrid systems because their solutions are

Manuscript received 21 January 2023; accepted 5 August 2023. Date of publication 6 September 2023; date of current version 19 December 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 62073166, Grant 62221004, Grant 62203214, and Grant 62176127; in part by the Basic Science (Natural Science) Research Project of Higher Education in Jiangsu Province under Grant 21KJB120006 and Grant 22KJB120003; and in part by the Jiangsu University Philosophy and Social Science Research Project under Grant 2021SJA0353. This article was recommended by Associate Editor D. Wang. *(Corresponding author: Shengyuan Xu.)*

Huihui Ji is with the School of Mathematics, Nanjing Audit University, Nanjing 211815, China (e-mail: jihuihui2020@nau.edu.cn).

Shengyuan Xu is with the School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China (e-mail: syxu@njust.edu.cn).

He Zhang is with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing 211800, China (e-mail: zhanghe@njtech.edu.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TSMC.2023.3305518.

Digital Object Identifier 10.1109/TSMC.2023.3305518

too complex to analyze directly in such case. Motivated by this, the problem of stochastic scheduling control on unknown nonlinear systems (UNSs) is studied.

B. Outline and Contribution

In [6], [7], [8], and [9], the scheduling process is realized by event-triggered schemes; Hu et al. [10] focused on the problem of preassigned-time synchronization and design an interesting control scheme with accurate estimates of the settling time; further, to improve the estimate of settling time, the fixed-time control scheme is proposed with reducing the chattering caused by the sign function [11]. Notice that the above works are only time scheduling. In this article, the studied UNSs are continuous-time with sampled-data inputs, where the resources are scheduled by allotted execution time and allotted controller gains. To overcome the difficulty resulted by the unknown nonlinearity, an approximation technique based on the neural network (NN) is introduced. Currently, the NN-based approach has been rapidly developed and successfully applied into control domain [12], [13]. It is always observed from a large number of simulation research and practical industrial implementation that NNs are very effective in handling unknown nonlinearity and the controller design of UNSs. Different from [4] and [5], the feedback control signal in this article is produced based on periodic sampled data, which is the other reason of analysis complexity of this work. As one of the most powerful analysis approaches, a delay system is introduced to solve the problem resulted by sampled data. The main highlights of this article are as follows.

1) A suitable Lyapunov analysis method is given to overcome the difficulties resulted by discrete-time inputs and nonlinearity, these difficulties are not solved by approaches given in [4] and [5].
2) Stability criteria are obtained based on the probability distribution of the random variable on the allotted process by using a general Lyapunov functional and some stochastic analysis techniques.

Notations: $\mathbb{R}^n$ is an n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is a set produced by all of real matrices with $n\times m$ dimensions, A^{T} is the transpose of matrix A, $A<0$ implies that matrix A is symmetric negative definite, $\mathrm{Sym}\{A\}=A+A^{\mathrm{T}}$, $\mathrm{diag}\{x,y,z\}=\begin{bmatrix} x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$, and $\|A\|$ is the norm of A.

II. Problem Formulation and Preliminaries

A. System Description

Consider a class of UNSs governed by the following ordinary differential equation:

$$\dot{x}(t)=Ax(t)+f(x(t),t)+Bu(t) \tag{1}$$

where $x\in\mathbb{R}^n$, $u\in\mathbb{R}^p$, and $f\in\mathbb{R}^n$ are the state variable, input variable, and continuous unknown nonlinear function of the system, respectively; and A and B are known matrices with appropriate dimensions.

B. NN-Based Approximation Model

For addressing the control problem of the UNS, the unknown nonlinear function f should be handled. Notice that NNs are widely studied and applied [14], [15], [16], [17], [18], [19] because their effectiveness has been verified in handling unknown nonlinearity and the controller design of UNSs. Motivated by this, a multilayer NN in this article is applied into learn the unknown nonlinear function.

Under zero bias terms, a multilayer NN, including one hidden layer, is considered. It is given in the forms of matrix–vector

$$f_{nn}(x(t),t,M_1,M_2)=M_2\varpi(M_1x(t)) \tag{2}$$

where $f_{nn}\in\mathbb{R}^n$ is the NN output; $M_1\in\mathbb{R}^{n_1\times n}$ is the first-to-second layer interconnection weight matrix; $M_2\in\mathbb{R}^{n\times n_1}$ is the second-to-third layer interconnection weight matrix; n_1 is the number of hidden neurons; and vector function $\varpi:\mathbb{R}^{n_1}\to\mathbb{R}^{n_1}$ is the activation function given as follows:

$$\varpi(y)=\begin{bmatrix}\varpi_1(y_1) & \varpi_2(y_2) & \cdots & \varpi_{n_1}(y_{n_1})\end{bmatrix}^{\mathrm{T}} \tag{3}$$

where $y=\begin{bmatrix}y_1 & y_2 & \cdots & y_{n_1}\end{bmatrix}^{\mathrm{T}}$, the activation function denoted by ϖ_i $(i=1,2,\ldots,n_1)$ is of the neuron i. Here, ϖ_i is assumed to obey the following conditions.

Assumption 1: Throughout this paper, the activation function $\overline{\varpi}_i$ satisfies the conditions as follows.

1) ϖ_i is smooth everywhere.
2) For $y_i\in\mathbb{R}$ and $q_i>0$, $-q_i\le\varpi_i(y_i)\le q_i$ and $\varpi_i(y_i)|_{y_i=0}=0$.

Based on Assumption 1, the bipolar sigmoid function given in [20], which is associated to the origin, is chosen by this article as follows:

$$\varpi_i(y_i)=\frac{q_i\left(1-e^{-y_i/d_i}\right)}{1+e^{-y_i/d_i}} \tag{4}$$

where q_i and d_i in the ith bipolar sigmoid function are two positive numbers. The key of approximating unknown nonlinear function is to train the NN (3) Using the backpropagation procedure, all connecting weights of the multilayer NN are determined by a learning rule.

Following the approximation theorem of multilayer NNs given in [21], with giving $\delta>0$, the desired parameters M_1^* and M_2^* can be obtained by

$$(M_1^*,M_2^*)=\arg\min_{(M_1,M_2)}\left\{\max_{x\in\mathcal{D}}\|f(x(t),t)-f(x(t),t,M_1,M_2)\|\right\}$$

such that

$$\max_{x\in\mathcal{D}}\|f(x(t),t)-f(x(t),t,M_1,M_2)\|\le\delta\|x(t)\| \tag{5}$$

in which $\mathcal{D}$ is a compact subset of $\mathbb{R}^n$.

Let $\dot{\varpi}_i(y_i)=([d\varpi_i(y_i)]/dy_i)$. Its boundary values are given as follows:

$$\phi_{i,m}=\min_{y_i}\dot{\varpi}_i(y_i),\quad \phi_{i,M}=\max_{y_i}\dot{\varpi}_i(y_i). \tag{6}$$

It is easy to see that

$$\frac{\varpi_i(y_i)}{y_i}\in\left[\phi_{i,m},\phi_{i,M}\right] \tag{7}$$

which implies that

$$\big(\varpi_i(y_i) - \phi_{i,m}y_i\big)\big(\varpi_i(y_i) - \phi_{i,M}y_i\big) \le 0. \tag{8}$$

From the bipolar sigmoid function (4), one has $\phi_{i,m} = 0$ and $\phi_{i,M} = (q_i/2d_i)$.

The following system approximated by NN (3) is derived:

$$\dot{x}(t) = Ax(t) + M_2^{*}\varpi\big(M_1^{*}x(t)\big) + \varepsilon(x(t)) + Bu(t) \tag{9}$$

where $\varepsilon(x(t)) = f(x(t), t) - M_2^{*}\varpi(M_1^{*}x(t))$.

C. Scheduling Control Scheme

In this article, a sampled-data-based scheduling control scheme is considered, where the sensor is periodic time-driven with a given sampling period T_c, the controller is event-driven and stochastic scheduling.

Let $\ell(t_k)$ be a discrete-time Markov process signal with a finite jump state set $\mathcal{L} = \{1, 2, \ldots, m_\ell\}$ and probability distribution $\mathcal{P} = [p_1 \cdots p_{m_\ell}]$ and $r(t_k)$ be also a discrete-time switching signal with finite switching state sets ${}^{[i_\ell]}\mathcal{R} = \{1, 2, \ldots, m_r^{[i_\ell]}\}$ and probability distribution ${}^{[i_\ell]}\wp = [{}^{[i_\ell]}\sigma_1 \; {}^{[i_\ell]}\sigma_2 \cdots {}^{[i_\ell]}\sigma_{m_r}]$ for $i_\ell \in \mathcal{L}$. ${}^{[i_\ell]}\Gamma_{i_r}$ be a scheduling control protocol, where there are i_ℓ (with $i_\ell \in \mathcal{L}$) controllers allotted sequentially, but the execution time of each allotted controller is given randomly, here, we choose the i_rth (with $i_r \in {}^{[i_\ell]}\mathcal{R}$) group of the execution time allotted the scheduler under random jump state i_ℓ. Let ${}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}$ be the execution time of the jth controller for $i_r \in {}^{[i_\ell]}\mathcal{R}$ under the random jump state $i_\ell \in \mathcal{L}$. Thus, consider the periodic sampling scheme and the sampling interval $[t_k, t_{k+1})$ can be divided into i_ℓ subintervals

$$\begin{aligned} {}^{[i_\ell]}\Gamma_{i_r} : &\Big[t_k, t_k + {}^{[i_\ell]}\mathcal{T}_{[1]}^{i_r}\Big) \\ &\Bigg[t_k + {}^{[i_\ell]}\mathcal{T}_{[1]}^{i_r}, t_k + \sum_{j=1}^{2} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\Bigg) \\ &\ldots, \Bigg[t_k + \sum_{j=1}^{i_\ell - 1} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}, t_k + \sum_{j=1}^{i_\ell} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\Bigg) \end{aligned}$$

where $i_\ell \in \mathcal{L}$, $i_r \in \mathcal{R}$, and $t_{k+1} = t_k + \sum_{j=1}^{i_\ell} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r})$.

In order to clearly show the logic of the random control mechanism, a simple example is introduced in Fig. 1. In view of the figure, there are three allotted controllers ($m_\ell = 3$), and two groups of the execution time allotted the scheduler under random jump state i_ℓ. The top subgraph shows the response of Markov jump variable $\ell(t)$ by the fine solid line and the subgraph below shows the response of random variable $r(t)$ by the big solid line. In addition, the choice method of three controllers is shown in the subgraph below, where the three shadow modules represent the execution times of three controllers, for example, take the interval segment $[t_2, t_3)$: ${}^{[3]}\mathcal{T}_{[1]}^{2}$ is the execution time of the 1st controller under Markov jump state $\ell(t) = 3$ and random jump state $r(t) = 2$; ${}^{[3]}\mathcal{T}_{[2]}^{2}$ is the execution time of the 2nd controller under Markov jump state $\ell(t) = 3$ and random jump state $r(t) = 2$; ${}^{[3]}\mathcal{T}_{[3]}^{2}$ is the execution time of the 3rd controller under Markov jump state $\ell(t) = 3$ and random jump state $r(t) = 2$.

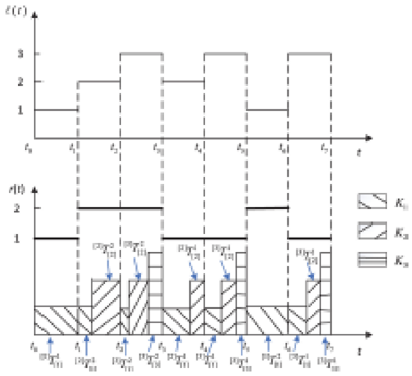


Fig. 1. Diagram of stochastic scheduling control.

In this article, for $\ell(t_k) = i_\ell, r(t_k) = i_r$, and $1 \le i_c \le i_\ell$, the following stochastic scheduling protocols are proposed:

$$\begin{aligned} {}^{i_\ell}\Gamma_{i_r} : u(t) &= K_{[i_c]}x(t_k) \\ t &\in \Bigg[t_k + \sum_{j=1}^{i_c - 1} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}, t_k + \sum_{j=1}^{i_c} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\Bigg) \end{aligned} \tag{10}$$

where it is defined that $\sum_{j=1}^{i_c - 1} {}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r} = 0$ for $i_c = 1$, $K_{[i_c]}$ is the controller gain under the i_cth allotted execution time interval.

It is considered that controllers on each group allotted execution time are executed sequentially, which means that the kth controller does not execute until all i_c controllers ($i_c < k$) have finished.

Remark 1: Notice that the Markov chain $\ell(t)$ and random switching signal $r(t)$ are aperiodic, irreducible, and ergodic, which implies that each jump mode of Markov chain $\ell(t)$ or random signal $r(t)$ occurs frequently as the variable t goes to large enough.

Remark 2: In this article, the occurrence of each jump mode of Markov chain $\ell(t)$ is not affected by random switching signal $r(t)$. Conversely, the occurrence of each jump mode of random switching signal $r(t)$ is governed by the Markov chain.

Let $\tau(t) = t - t_k$ for $t \in [t_k, t_{k+1})$ and $[t_k, t_{k+1}) = \cup_{i_c=1}^{\ell(t_k)} {}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)}$, where ${}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)} = [t_k + \sum_{j=1}^{i_c - 1} {}^{[i_\ell]}\mathcal{T}_{[j]}^{r(t_k)}, t_k + \sum_{j=1}^{i_c} {}^{[i_\ell]}\mathcal{T}_{[j]}^{r(t_k)})$. It is clear that $0 \le \tau(t) \le T_c$, where T_c stands for the sampling period. Based on the above analysis, the following closed-loop system with scheduling controllers is obtained:

$$\begin{aligned} \dot{x}(t) = Ax(t) &+ M_2^{*}\varpi\big(M_1^{*}x(t)\big) + \varepsilon(t) \\ &+ BK_{[i_c]}x(t - \tau(t)) \\ t &\in {}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)}, \; i_c \le \ell(t_k). \end{aligned} \tag{11}$$

In what follows, almost sure stability of the system (11) will be discussed.

Definition 1 [22]: The stochastic system (11) is almost surely (a.s.) exponentially stable, if there exists a number $\beta > 0$ such that for any initial condition one has

$$\mathcal{P}\left\{\lim_{t\to\infty}\sup\frac{1}{t}\ln\|x(t)\| \le -\beta\right\} = 1.$$

Remark 3: It is worth mentioning that if there exists a positive real number $\beta > 0$ such that $\ln\|x(t)\| \le -\beta t$ holds, then one can obtain $\|x(t)\| \le e^{-\beta t}$, which implies that $\|x(t)\|$ exponentially converges to zero as the variable t goes to infinity. Then the system is exponentially stable. Similarly, if there exists a positive real number $\beta > 0$ such that $\mathcal{P}\{\ln\|x(t)\| \le -\beta t\} = 1$, that is to say, $\ln\|x(t)\| \le -\beta t$ holds with probability 1, which implies that the system is a.s. exponentially stable.

III. Main Results

A. Stability Conditions of the Closed-Loop System

Theorem 1: Let $V(t)$ be Lyapunov functional along the closed-loop system (11) on sampling interval $[t_k, t_{k+1}) = \cup_{i_c=1}^{\ell(t_k)}\, {}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)}$, where ${}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)} = [t_k + \sum_{j=1}^{i_c-1} {}^{[i_\ell]}\mathcal{T}_{[j]}^{r(t_k)}, t_k + \sum_{j=1}^{i_c} {}^{[i_\ell]}\mathcal{T}_{[j]}^{r(t_k)})$. If there exist m real numbers $\alpha_{[i]}$ $(i = 1, 2, \ldots, m)$, real numbers $\beta > 0$, $\bar{\beta} > 0$, and a symmetric positive-definite matrix Q such that

$$\beta x(t)^{\mathrm{T}} Qx(t) \le V(t) \le \bar{\beta} x(t)^{\mathrm{T}} Qx(t) \tag{12}$$

$$\dot{V}(t) \le \alpha_\ell^{[\ell(t_k)]}\alpha_{i_c} x(t)^{\mathrm{T}} Qx(t),\ \ t \in {}^{[\ell(t_k)]}\mathbf{I}_{[i_c]}^{r(t_k)} \tag{13}$$

$$\ln\left(\frac{\bar{\beta}}{\beta}\right)\sum_{i_\ell=1}^{m_\ell} i_\ell p_{i_\ell} + \sum_{i_\ell=1}^{m_\ell} p_{i_\ell}\alpha_\ell^{[i_\ell]}\sum_{j=1}^{i_\ell}\sum_{i_r=1}^{m_r}\sigma_{i_r}\alpha_{[j]}\left({}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\right) < 0 \tag{14}$$

then the stochastic scheduling scheme can keep a.s. exponential stability of the closed-loop system (11).

Proof: For $t \in {}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)}$, from the given conditions (12) and (13), it is derived based on Gronwall's inequality that

$$\begin{aligned}
&x\left(t_k + \sum_{j=1}^{i_c} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)^{\mathrm{T}} Qx\left(t_k + \sum_{j=1}^{i_c} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)\\
&\le \frac{\bar{\beta}}{\beta} e^{\frac{\left(\alpha_\ell^{[\ell(t_k)]}\alpha_{[i_c]}\right)\left({}^{[\ell(t_k)]}\mathcal{T}_{[i_c]}^{r(t_k)}\right)}{\beta}} x\left(t_k + \sum_{j=1}^{i_c-1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)^{\mathrm{T}}\\
&\quad \times Qx\left(t_k + \sum_{j=1}^{i_c-1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right).
\end{aligned} \tag{15}$$

Since $V(t)$ is a Lyapunov functional along the closed-loop system (11) on sampling interval $[t_k, t_{k+1})$, it is continuous on the time interval. For $t \in {}^{[i_\ell]}\mathbf{I}_{[i_c+1]}^{r(t_k)}$, from the given condition (13), one has

$$\begin{aligned}
&x\left(t_k + \sum_{j=1}^{i_c+1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)^{\mathrm{T}} Qx\left(t_k + \sum_{j=1}^{i_c+1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)\\
&\le \frac{\bar{\beta}}{\beta} e^{\frac{\left(\alpha_\ell^{[\ell(t_k)]}\alpha_{[i_c+1]}\right)\left({}^{[\ell(t_k)]}\mathcal{T}_{[i_c+1]}^{r(t_k)}\right)}{\beta}} x\left(t_k + \sum_{j=1}^{i_c} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)^{\mathrm{T}}\\
&\quad \times Qx\left(t_k + \sum_{j=1}^{i_c} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)\\
&\le \left(\frac{\bar{\beta}}{\beta}\right)^2 e^{\frac{\left(\alpha_\ell^{[\ell(t_k)]}\right)\sum_{j=i_c}^{i_c+1}\alpha_{[j]}\left({}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)}{\beta}} x\left(t_k + \sum_{j=1}^{i_c-1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)^{\mathrm{T}}\\
&\quad \times Qx\left(t_k + \sum_{j=1}^{i_c-1} {}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right).
\end{aligned} \tag{16}$$

Thus, for $t \in [t_k, t_{k+1})$, one has

$$\begin{aligned}
&x(t_{k+1})^{\mathrm{T}} Qx(t_{k+1})\\
&\le \left(\frac{\bar{\beta}}{\beta}\right)^{\ell(t_k)} e^{\frac{\left(\alpha_\ell^{[\ell(t_k)]}\right)\sum_{j=1}^{\ell(t_k)}\alpha_{[j]}\left({}^{[\ell(t_k)]}\mathcal{T}_{[j]}^{r(t_k)}\right)}{\beta}} x(t_k)^{\mathrm{T}} Qx(t_k)
\end{aligned} \tag{17}$$

which implies that for $t \in [t_0, t_{k+1})$

$$\begin{aligned}
x(t_{k+1})^{\mathrm{T}} Qx(t_{k+1}) &\le \gamma_1 x(t_{k-1})^{\mathrm{T}} Qx(t_{k-1})\\
&\le \gamma_2 x(t_0)^{\mathrm{T}} Qx(t_0) = \gamma_3 x(t_0)^{\mathrm{T}} Qx(t_0)
\end{aligned} \tag{18}$$

where

$$\gamma_1 = \left(\frac{\bar{\beta}}{\beta}\right)^{\sum_{\theta=t_{k-1}}^{t_k}\ell(\theta)} e^{\frac{\sum_{\theta=t_{k-1}}^{t_k}\left(\alpha_\ell^{[\ell(\theta)]}\right)\sum_{j=1}^{\ell(\theta)}\alpha_{[j]}\left({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}\right)}{\beta}}$$

$$\gamma_2 = \left(\frac{\bar{\beta}}{\beta}\right)^{\sum_{\theta=t_0}^{t_k}\ell(\theta)} e^{\frac{\sum_{\theta=t_0}^{t_k}\left(\alpha_\ell^{[\ell(\theta)]}\right)\sum_{j=1}^{\ell(\theta)}\alpha_{[j]}\left({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}\right)}{\beta}}$$

$$\gamma_3 = e^{\ln\left(\frac{\bar{\beta}}{\beta}\right)\sum_{\theta=t_0}^{t_k}\ell(\theta) + \frac{\sum_{\theta=t_0}^{t_k}\left(\alpha_\ell^{[\ell(\theta)]}\right)\sum_{j=1}^{\ell(\theta)}\alpha_{[j]}\left({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}\right)}{\beta}}.$$

The following work is to verify $\gamma_3 < 1$ in (18), which can guarantee the closed-loop system is stable.

Let $N_{[i_\ell]}$ be the number of random variable $\ell(\theta) = i_\ell$ and $N_{[i_\ell],i_r}$ be number of random variable $r(\theta) = i_r$ for $\theta \in [t_0, t_k]$, $i_\ell \in \mathcal{L}$, and $i_r \in \mathcal{R}$. It is clear that

$$\sum_{i_\ell=1}^{m_\ell} N_{[i_\ell]} = k+1$$

$$\sum_{i_\ell=1}^{m_\ell}\sum_{i_r=m_r}^{m_\ell} N_{[i_\ell],i_r} = k+1.$$

Without loss of generality, the random sequence $\{Y(\theta, [\ell(\theta)], r(\theta)) = (\alpha_\ell^{[\ell(\theta)]})\sum_{j=1}^{\ell(\theta)}\alpha_{[j]}({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}),\ \theta = t_0, \ldots, t_k\}$ is reorganized and reordered as follows:

$$\left\{Y(\theta, [1], r(\theta))|\ \theta = s_1^{[1]}, s_2^{[1]}, \ldots, s_{N_{[1]}}^{[1]}\right\}$$
$$\left\{Y(\theta, [2], r(\theta))|\ \theta = s_1^{[2]}, s_2^{[2]}, \ldots, s_{N_{[2]}}^{[2]}\right\}$$
$$\vdots$$
$$\left\{Y(\theta, [m_\ell], r(\theta))|\ \theta = s_1^{[m_\ell]}, s_2^{[m_\ell]}, \ldots, s_{N_{[m_\ell]}}^{[m_\ell]}\right\}.$$

Moreover, the random sequence $\{Y(\theta, [i_\ell], r(\theta))|\ \theta = s_0^{[i_\ell]}, s_1^{[i_\ell]}, \ldots, s_{N_{[i_\ell]}}^{[i_\ell]}\}$, for $i_\ell \in \mathcal{L}$, is reorganized and reordered as follows:

$$\left\{Y\left(\theta, [i_\ell], 1|\ \theta = n_1^{[i_\ell]}, n_2^{[i_\ell]}, \ldots, n_{N_{[i_\ell],1}}^{[i_\ell]}\right\}\right.$$
$$\left\{Y\left(\theta, [i_\ell], 2|\ \theta = n_1^{[i_\ell]}, n_2^{[i_\ell]}, \ldots, n_{N_{[i_\ell],2}}^{[i_\ell]}\right\}\right.$$
$$\vdots$$
$$\left\{Y\left(\theta, [i_\ell], m_r|\ \theta = n_1^{[i_\ell]}, n_2^{[i_\ell]}, \ldots, n_{N_{[i_\ell],m_r}}^{[i_\ell]}\right\}\right..$$

Based on the above analysis, it is easy to see that $N_{i_\ell} \to \infty$ and $N_{[i_\ell],i_r} \to \infty$ hold for $i_\ell \in \mathcal{L}$, $i_r \in {}^{[i_\ell]}\mathcal{R}$ as $k \to \infty$ for the sequence $\{t_k, k \geq 0\}$. Thus, it is derived that

$$\begin{aligned}
&\lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=t_0}^{t_k} \left(\alpha_\ell^{[\ell(\theta)]}\right) \sum_{j=1}^{\ell(\theta)} \alpha_{[j]}\left({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}\right) \\
&= \lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=s_1^{[1]}}^{s_{N_{[1]}}^{[1]}} Y(\theta, [1], r(\theta)) \\
&\quad + \lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=s_{[1]}^{[2]}}^{s_{N_{[2]}}^{[2]}} Y(\theta, [2], r(\theta)) \\
&\quad \vdots \\
&\quad + \lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=s_{[1]}^{[m_\ell]}}^{s_{N_{[m_\ell]}}^{[m_\ell]}} Y(\theta, [m_\ell], r(\theta)). \qquad (19)
\end{aligned}$$

Based on the strong law of large numbers, it is clear that

$$\lim_{k\to\infty} \frac{N_{[i_\ell]}}{k+1} = p_\ell, \quad \lim_{N_{i_\ell}\to\infty} \frac{N_{[i_\ell],i_r}}{N_{[i_\ell]}} = {}^{[i_\ell]}\sigma_\ell \qquad (20)$$

$$\frac{\sum_{\theta=s_1^{[1]}}^{s_{N_{[1]}}^{[1]}} Y(\theta, [1], i_r)}{N_{[1],i_r}} = \alpha_\ell^{[1]} \alpha_{[1]}\left({}^{[1]}\mathcal{T}_{[1]}^{i_r}\right). \qquad (21)$$

Thus, one has

$$\begin{aligned}
&\lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=s_1^{[1]}}^{s_{N_{[1]}}^{[1]}} Y(\theta, [1], r(\theta)) \\
&= \lim_{k\to\infty} \left(\frac{N_{[1]}}{k+1} \times \frac{\sum_{\theta=s_1^{[1]}}^{s_{N_{[1]}}^{[1]}} Y(\theta, [1], r(\theta))}{N_{[1]}} \right) \\
&= \lim_{k\to\infty} \frac{N_{[1]}}{k+1} \times \sum_{i_r=1}^{m_r} \left(\frac{N_{[1],i_r}}{N_{[1]}} \times \frac{\sum_{\theta=s_1^{[1]}}^{s_{N_{[1]}}^{[1]}} Y(\theta, [1], i_r)}{N_{[1],i_r}} \right) \\
&= \lim_{k\to\infty} \left(\frac{N_1}{k+1} \right) \times \sum_{i_r=1}^{m_r} \alpha_\ell^{[1]} \alpha_{[1]}\left({}^{[1]}\mathcal{T}_{[1]}^{i_r}\right) \left(\lim_{N_{[1]}\to\infty} \frac{N_{[1],i_r}}{N_{[1]}} \right) \\
&= p_1{}^{[1]}\alpha_\ell^{[1]} \sum_{i_r=1}^{m_r} \alpha_{[1]}\left({}^{[1]}\mathcal{T}_{[1]}^{i_r}\right)\sigma_{i_r}. \qquad (22)
\end{aligned}$$

Therefore, similar to analysis (22), one has

$$\begin{aligned}
&\lim_{k\to\infty} \frac{1}{k+1} \sum_{\theta=t_0}^{t_k} \left(\alpha_\ell^{[\ell(\theta)]}\right) \sum_{j=1}^{\ell(\theta)} \alpha_{[j]}\left({}^{[\ell(\theta)]}\mathcal{T}_{[j]}^{r(\theta)}\right) \\
&= \sum_{i_\ell=1}^{m_\ell} p_{i_\ell} \alpha_\ell^{[i_\ell]} \sum_{i_r=1}^{m_r} \sum_{j=1}^{i_\ell} \alpha_{[j]}\left({}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\right)\sigma_{i_r}. \qquad (23)
\end{aligned}$$

In addition, using the strong law of large numbers, one has

$$\lim_{k\to\infty} \frac{1}{k+1} \ln\left(\frac{\bar{\beta}}{\beta}\right) \sum_{\theta=t_0}^{t_k} \ell(\theta) = \ln\left(\frac{\bar{\beta}}{\beta}\right) \sum_{i_\ell=1}^{m_\ell} i_\ell p_{i_\ell}. \qquad (24)$$

Based on the given condition (14) $\ln(\bar{\beta}/\beta) \sum_{i_\ell=1}^{m_\ell} i_\ell p_{i_\ell} + \sum_{i_\ell=1}^{m_\ell} p_{i_\ell} \alpha_\ell^{[i_\ell]} \sum_{i_r=1}^{m_r} \sum_{j=1}^{i_\ell} \alpha_{[j]}({}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r})\sigma_{i_r} < 0$, one has that $x(t_{k+1})^{\mathrm{T}} Q x(t_{k+1})$ exponentially converges to zero point, which implies that conditions (12)–(14) can guarantee a.s. exponential stability of the closed system (11). ■

B. Design of Controller Gains

In the following, a design method of scheduling controller gains is given according to Theorem 1. Let us start with the following two lemmas.

Lemma 1 [23, Corollary 1]: Let $x(t)$ be a differentiable function: $[a, b] \to \mathbb{R}^n$. For given integers $m, N \in \mathbb{N}$ satisfying $m \geq N$, matrices $R \in \mathbb{S}_n^+$ and $M_i \in \mathbb{R}^{5n\times n}$, $i \in \{0, 1, 2, 3\}$, the following inequality

$$-\int_a^b \dot{x}(s)^{\mathrm{T}} R \dot{x}(s) ds \leq \vartheta^{\mathrm{T}} \Phi \vartheta \qquad (25)$$

holds, where

$$\begin{aligned}
\vartheta &= \left[x(b)^{\mathrm{T}}\ x(a)^{\mathrm{T}}\ \tfrac{\Omega_1}{b-a}\ \tfrac{2\Omega_2}{(b-a)^2}\ \tfrac{6\Omega_3}{(b-a)^3} \right] \\
\Omega_1 &= \int_a^b x(s)ds, \quad \Omega_2 = \int_a^b \int_{s_1}^b x(s_2)ds_2 ds_1 \\
\Omega_3 &= \int_a^b \int_{s_1}^b \int_{s_2}^b x(s_3)ds_3 ds_2 ds_1 \\
\Phi &= \sum_{i=0}^{3} \frac{b-a}{2k+1} M_i R^{-1} M_i^{\mathrm{T}} + \sum_{i=0}^{3} \left[M_i \Pi_i + (M_i \Pi_i)^{\mathrm{T}} \right] \\
\Pi_0 &= e_1 - e_2, \quad \Pi_1 = e_1 + e_2 - 2e_3 \\
\Pi_2 &= e_1 - e_2 + 6e_3 - 6e_4 \\
\Pi_3 &= e_1 + e_2 - 12e_3 + 30e_4 - 20e_5 \\
e_i &= \left[0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(5-i)n} \right], i = 1, 2, \ldots, 5.
\end{aligned}$$

Lemma 2 [24], [25]: For any vector $\xi \in \mathbb{R}^m$, matrices $R_1, R_2 \in \mathbb{S}_n^+$, $S \in \mathbb{R}^{n\times n}$, $W_1, W_2 \in \mathbb{R}^{n\times n}$, and real scalars $\alpha \geq 0$, $\beta \geq 0$ satisfying $\alpha + \beta = 1$, the following inequality holds:

$$\begin{aligned}
&\frac{1}{\alpha} \xi^{\mathrm{T}} W_1^{\mathrm{T}} R_1 W_1 \xi + \frac{1}{\beta} \xi^{\mathrm{T}} W_2^{\mathrm{T}} R_2 W_2 \xi \\
&\geq \xi^{\mathrm{T}} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & S \\ S^{\mathrm{T}} & R_2 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \xi.
\end{aligned}$$

Theorem 2: Consider the closed-loop system (11) with initial state $x(t) = 0$ for $t < t_0$ and the sampling interval $[t_k, t_{k+1}) = \cup_{i_c=1}^{\ell(t_k)\,[i_\ell]}\mathbb{I}_{[i_c]}^{r(t_k)}$ defined in Theorem 1. Give the execution time length $({}^{[i_\ell]}\mathcal{T}_{[i_c]}^{i_r})$, real numbers $\beta > 0$, $\bar{\beta} > 0$, matrices $W_1^{[i_c]}$, $W_2^{[i_c]}$. The closed-loop system (11) is a.s. exponentially stable under scheduling controllers if there exist m real number $\alpha_{[i_c]}$ and $\gamma_{[i_c]} > 0$, symmetric positive-definite matrices Q, R_1, R_2, R_3, R_4, $P_{[i_c]} =$

$\{p_1^{[i_c]}, \ldots, p_n^{[i_c]}\}$, and matrices $S_{[i_c]}$ ($i_c \in \mathcal{L}$) such that for $i_\ell \in \mathcal{L}$

$$\underline{\beta} Q \leq R_1 \leq \bar{\beta} Q \tag{26}$$

$$\begin{bmatrix} \Phi_1^{[i_c]} & \Phi_2^{[i_c]} \\ \left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} & \Phi_3^{[i_c]} \end{bmatrix} \leq 0 \quad i = 1, 2, \ldots, m \tag{27}$$

$$\begin{bmatrix} \Psi_1^{[i_c]} & \Phi_2^{[i_c]} \\ \left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} & \Phi_3^{[i_c]} \end{bmatrix} \leq 0 \quad i = 1, 2, \ldots, m \tag{28}$$

$$\ln\left(\frac{\bar{\beta}}{\underline{\beta}}\right) \sum_{i_\ell=1}^{m_\ell} i_\ell p_{i_\ell} + \sum_{i_\ell=1}^{m_\ell} p_{i_\ell} \alpha_\ell^{[i_\ell]} \sum_{i_r=1}^{m_r} \sigma_{i_r} \sum_{j=1}^{i_\ell} \alpha_{[j]} \left({}^{[i_\ell]}\mathcal{T}_{[j]}^{i_r}\right) < 0 \tag{29}$$

where $\mathbb{e}_l = \left[0_{n\times(l-1)n}\ I_n\ 0_{n\times(9-l)n}\right], l = 1, 2, \ldots, 10$

$$\begin{aligned}
\Phi_1^{[i_c]} = {} & \mathrm{Sym}\{\mathcal{A}_{[i_c]}^{\mathrm{T}} R_1 \mathbb{e}_1\} + \mathbb{e}_1^{\mathrm{T}}\left(R_2 - \alpha_\ell^{[i_\ell]}\alpha_{[i_c]} Q\right)\mathbb{e}_1 \\
& - \mathbb{e}_5^{\mathrm{T}} R_2 \mathbb{e}_5 + \mathbb{e}_{10}^{\mathrm{T}}(R_3 + T_c R_4)\mathbb{e}_{10} \\
& - \frac{\pi^2}{4T_c^2}(\mathbb{e}_1 - \mathbb{e}_4)^{\mathrm{T}} R_3(\mathbb{e}_1 - \mathbb{e}_4) + \mathrm{Sym}\left\{M_0^{[i_c]}(\mathbb{e}_1 - \mathbb{e}_4)\right\} \\
& + \mathrm{Sym}\left\{M_1^{[i_c]}(\mathbb{e}_1 + \mathbb{e}_4 - 2\mathbb{e}_6)\right\} \\
& + \mathrm{Sym}\left\{M_2^{[i_c]}(\mathbb{e}_1 - \mathbb{e}_4 + 6\mathbb{e}_6 - 6\mathbb{e}_7)\right\} \\
& + \mathrm{Sym}\left\{\hat{M}_0^{[i_c]}(\mathbb{e}_4 - \mathbb{e}_5)\right\} + \mathrm{Sym}\left\{\hat{M}_1^{[i_c]}(\mathbb{e}_4 + \mathbb{e}_5 - 2\mathbb{e}_8)\right\} \\
& + \mathrm{Sym}\left\{\hat{M}_2^{[i_c]}(\mathbb{e}_4 - \mathbb{e}_5 + 6\mathbb{e}_8 - 6\mathbb{e}_9)\right\} \\
& - \mathrm{Sym}\left\{(\mathbb{e}_2 - \Xi_{\min} M_1^{*}\mathbb{e}_1)^{\mathrm{T}} P_{[i_c]}(\mathbb{e}_2 - \Xi_{\max} M_1^{*}\mathbb{e}_1)\right\} \\
& + \gamma_{[i_c]}\left(\delta^2 \mathbb{e}_1^{\mathrm{T}}\mathbb{e}_1 - \mathbb{e}_3^{\mathrm{T}}\mathbb{e}_3\right) \\
& + \mathrm{Sym}\left\{\left(W_1^{[i_c]}\mathbb{e}_1 + W_2^{[i_c]}\mathbb{e}_{10}\right)^{\mathrm{T}}(\mathcal{A}_{[i_c]} - \mathbb{e}_{10})\right\}
\end{aligned}$$

$$\Psi_1^{[i_c]} = \Phi_1^{[i_c]} - \mathrm{Sym}\{\mathcal{A}_{[i_c]}^{\mathrm{T}} \bar{\beta} Q \mathbb{e}_1\}$$

$$\mathcal{A}_{[i_c]} = A\mathbb{e}_1 + M_2^{*}\mathbb{e}_2 + \mathbb{e}_3 + BK_{[i_c]}\mathbb{e}_4$$

$$\Phi_2^{[i_c]} = \left[M_0^{[i_c]}\ M_1^{[i_c]}\ M_2^{[i_c]}\ \hat{M}_0^{[i_c]}\ \hat{M}_1^{[i_c]}\ \hat{M}_2^{[i_c]}\right]$$

$$\Phi_3^{[i_c]} = \begin{bmatrix} \Phi_{3a} & S_{[i_c]} \\ S_{[i_c]}^{\mathrm{T}} & \Phi_{3b} \end{bmatrix}, \ \Phi_{3a} = \mathrm{diag}\left\{\frac{-1}{T_c}R_4, \frac{-3}{T_c}R_4, \frac{-5}{T_c}R_4\right\}$$

$$\Phi_{3b} = \mathrm{diag}\left\{\frac{-1}{T_c}R_4, \frac{-3}{T_c}R_4, \frac{-5}{T_c}R_4\right\}.$$

Proof: Let $t_\tau = t - \tau(t)$. Construct the following Lyapunov functional along the closed-loop systems (11) on $[t_k, t_{k+1})$:

$$\begin{aligned}
V(t) = {} & x(t)^{\mathrm{T}} R_1 x(t) + \int_{t-T_c}^{t} x(s)^{\mathrm{T}} R_2 x(s) ds \\
& + \int_{t_k}^{t} \dot{x}(s)^{\mathrm{T}} R_3 \dot{x}(s) ds \\
& - \frac{\pi^2}{4T_c^2}\int_{t_\tau}^{t}(x(s) - x(t_\tau))^{\mathrm{T}} R_3 (x(s) - x(t_\tau)) ds \\
& + \int_{t-T_c}^{t}\int_{s_1}^{t} \dot{x}(s_2)^{\mathrm{T}} R_4 \dot{x}(s_2) ds_2 ds_1
\end{aligned} \tag{30}$$

where R_1, R_2, R_3, and R_4 are positive symmetric matrices with appropriate dimensions. Based on Wirtinger's inequality [26], one has

$$\begin{aligned}
0 \leq {} & \int_{t_\tau}^{t} \dot{x}(s)^{\mathrm{T}} R_3 \dot{x}(s) ds - \frac{\pi^2}{4T_c^2}\int_{t_\tau}^{t}(x(s) - x(t_\tau))^{\mathrm{T}} \\
& \times R_3(x(s) - x(t_\tau)) ds.
\end{aligned}$$

Taking the derivative of $V(t)$ on $t \in {}^{[i_\ell]}\mathbf{I}_{[i_c]}^{r(t_k)}$ defined in Theorem 1, one has that

$$\begin{aligned}
\dot{V}(t) = {} & 2\dot{x}(t)^{\mathrm{T}} R_1 x(t) + x(t)^{\mathrm{T}} R_2 x(t) \\
& - x(t - T_c)^{\mathrm{T}} R_2 x(t - T_c) + \dot{x}(t)^{\mathrm{T}}(R_3 + T_c R_4)\dot{x}(t) \\
& - \frac{\pi^2}{4T_c^2}(x(t) - x(t_\tau))^{\mathrm{T}} R_3 (x(t) - x(t_\tau)) \\
& - \int_{t-T_c}^{t} \dot{x}(s)^{\mathrm{T}} R_4 \dot{x}(s) ds_2.
\end{aligned} \tag{31}$$

Based on Lemma 1, it is easy to see that

$$\begin{aligned}
& - \int_{t-T_c}^{t} \dot{x}(s)^{\mathrm{T}} R_4 \dot{x}(s) ds_2 \\
& = - \int_{t_\tau}^{t} \dot{x}(s)^{\mathrm{T}} R_4 \dot{x}(s) ds_2 - \int_{t-T_c}^{t_\tau} \dot{x}(s)^{\mathrm{T}} R_4 \dot{x}(s) ds_2 \\
& \leq \vartheta_1^{\mathrm{T}} \Psi_1^{[i_c]} \vartheta_1 + \vartheta_2^{\mathrm{T}} \Psi_2^{[i_c]} \vartheta_2
\end{aligned} \tag{32}$$

where $\vartheta_{41} = ([2\int_{t_\tau}^{t}\int_{u_1}^{t} x(u_2) du_2 du_1]/\tau(t)^2)$ and

$$\vartheta_1 = \begin{bmatrix} x(t) \\ x(t_\tau) \\ \frac{\int_{t_\tau}^{t} x(u_1) du_1}{\tau(t)} \\ \vartheta_{41} \end{bmatrix} \vartheta_2 = \begin{bmatrix} x(t_\tau) \\ x(t - T_c) \\ \frac{\int_{t-T_c}^{t_\tau} x(u_1) du_1}{T_c - \tau(t)} \\ \frac{2\int_{t-T_c}^{t_\tau}\int_{u_1}^{t} x(u_2) du_2 du_1}{(T_c - \tau(t))^2} \end{bmatrix}$$

$$\Psi_1 = \sum_{l=0}^{2} \frac{\tau(t) M_l^{[i_c]} R_4^{-1}\left(M_l^{[i_c]}\right)^{\mathrm{T}}}{2k+1} + \sum_{l=0}^{2}\mathrm{Sym}\left\{M_l^{[i_c]}\Pi_l\right\}$$

$$\begin{aligned}
\Psi_2 = {} & \sum_{l=0}^{2} \frac{(T_c - \tau(t))\hat{M}_l^{[i_c]} R_4^{-1}\left(\hat{M}_l^{[i_c]}\right)^{\mathrm{T}}}{2k+1} \\
& + \sum_{i=0}^{2}\mathrm{Sym}\left\{\hat{M}_l^{[i_c]}\Pi_l\right\}, \ \Pi_0 = e_1 - e_2
\end{aligned}$$

$$\Pi_1 = e_1 + e_2 - 2e_3, \ \Pi_2 = e_1 - e_2 + 6e_3 - 6e_4$$

$$e_i = \left[0_{n\times(i-1)n}\ I_n\ 0_{n\times(4-i)n}\right], i = 1, 2, 3, 4.$$

In view of conditions (5) and (8), one has

$$\begin{aligned}
& \left(\varpi\left(M_1^{*}x(t)\right) - \Xi_{\min} M_1^{*}x(t)\right)^{\mathrm{T}} P_{[i_c]} \\
& \quad \times \left(\varpi\left(M_1^{*}x(t)\right) - \Xi_{\max} M_1^{*}x(t)\right) \leq 0
\end{aligned} \tag{33}$$

$$\gamma_{[i_c]}\left(\delta^2 x(t)^{\mathrm{T}} x(t) - \varepsilon(x(t))^{\mathrm{T}}\varepsilon(x(t))\right) \geq 0 \tag{34}$$

where $P_{[i_c]} = \mathrm{diag}\{p_1^{[i_c]}, p_2^{[i_c]}, \ldots, p_n^{[i_c]}\} > 0$ and $\gamma_{[i_c]} > 0$.

Based on the closed-loop system (11), for matrices $W_1^{[i_c]}$ and $W_2^{[i_c]}$, one has

$$\begin{aligned}
& \left(W_1^{[i_c]}x(t) + W_2^{[i_c]}\dot{x}(t)\right)^{\mathrm{T}}\left(Ax(t) + M_2^{*}\mu(M_1^{*}x(t))\right. \\
& \quad \left. + \epsilon(t) + BK_{[i_c]}x(t_\tau) - \dot{x}(t)\right) = 0.
\end{aligned} \tag{35}$$

Based on the above analysis, one has

$$\dot{V}(t) \le \alpha_\ell^{[\ell(t_k)]}\alpha_{[i_c]}x(t)^{\mathrm{T}}Qx(t) + \eta(t)^{\mathrm{T}}\begin{bmatrix} \Phi_1^{[i_c]} & \left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} \\ \Phi_2^{i_c} & \tilde{\Phi}_3^{i_c} \end{bmatrix}\eta(t) \tag{36}$$

where $\tilde{\Phi}_3 = \begin{bmatrix} \tilde{\Phi}_{3a} & 0 \\ 0 & \tilde{\Phi}_{3b} \end{bmatrix}$, and

$$\tilde{\Phi}_{3a} = \mathrm{diag}\left\{-\frac{1}{\tau(t)}R_4, -\frac{3}{\tau(t)}R_4, -\frac{5}{\tau(t)}R_4\right\}$$

$$\tilde{\Phi}_{3b} = \mathrm{diag}\left\{\frac{-1}{T_c-\tau(t)}R_4, \frac{-3}{T_c-\tau(t)}R_4, \frac{-5}{T_c-\tau(t)}R_4\right\}$$

$$\eta(t) = \Big[x(t)^{\mathrm{T}}\ \varpi\left(M_1^* x(t)\right)^{\mathrm{T}}\ \varepsilon(t)^{\mathrm{T}}\ x(t_\tau)^{\mathrm{T}}\ x(t-T_c)^{\mathrm{T}}\ \frac{\int_{t_\tau}^{t} x(u_1)^{\mathrm{T}}du_1}{\tau(t)}\ \frac{2\int_{t_\tau}^{t}\int_{u_1}^{t}\dot{x}(u_2)^{\mathrm{T}}du_2du_1}{\tau(t)^2}\ \frac{\int_{t-T_c}^{t_\tau} x(u_1)^{\mathrm{T}}du_1}{T_c-\tau(t)}\ \frac{2\int_{t-T_c}^{t_\tau}\int_{u_1}^{t}\dot{x}(u_2)^{\mathrm{T}}du_2du_1}{(T_c-\tau(t))^2}\ \dot{x}(t)^{\mathrm{T}}\Big].$$

Since

$$\tilde{\Phi}_{3a} = \frac{1}{T_c}\mathrm{diag}\left\{-\frac{R_4}{\frac{\tau(t)}{T_c}}, -\frac{3R_4}{\frac{\tau(t)}{T_c}}, -\frac{5R_4}{\frac{\tau(t)}{T_c}}\right\}$$

$$\tilde{\Phi}_{3b} = \frac{1}{T_c}\mathrm{diag}\left\{\frac{-R_4}{\frac{T_c-\tau(t)}{T_c}}, \frac{-3R_4}{\frac{T_c-\tau(t)}{T_c}}, \frac{-5R_4}{\frac{T_c-\tau(t)}{T_c}}\right\}$$

applying Schur's complement into the condition (27), one has

$$0 > \Phi_3^{[i_c]} - \Phi_2^{[i_c]}\left(\Phi_1^{[i_c]}\right)^{-1}\left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} \tag{37}$$

$$\ge \tilde{\Phi}_3 - \Phi_2^{[i_c]}\left(\Phi_1^{[i_c]}\right)^{-1}\left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} \tag{38}$$

where the last inequality is from Lemma 2. Applying Schur's complement into the last inequality, one has

$$\begin{bmatrix} \Phi_1^{[i_c]} & \left(\Phi_2^{[i_c]}\right)^{\mathrm{T}} \\ \Phi_2^{[i_c]} & \tilde{\Phi}_3 \end{bmatrix} < 0 \tag{39}$$

which implies that conditions (26)–(29) can guarantee

$$\dot{V}(t) \le \alpha_\ell^{[\ell(t_k)]}\alpha_{[i_c]}x(t)^{\mathrm{T}}Qx(t). \tag{40}$$

In addition, we should verify $\bar{V}(t) = V(t) - \bar{\beta}x(t)^{\mathrm{T}}Qx(t) \le 0$ to ensure the choice of $V(t)$ satisfies the condition (12). It is easy to see that the condition (28) can guarantee $\dot{\bar{V}}(t) \le 0$. Since $x(t) = 0$ for $t < t_0$, one has $V(t_0) = x(t_0)^{\mathrm{T}}R_1x(t_0)$ and $\bar{V}(t) \le \bar{V}(t_0) \le 0$.

Therefore, according to Theorem 1, the closed-loop system (11) is a.s. exponentially stable under stochastic scheduling controllers. ■

Remark 4: In view of $\Phi_1^{[i_c]}$ in Theorem 2, it is easy to see that the controller gains $K_{[i_c]}$ are coupled with auxiliary variables R_1, $W_1^{[i_c]}$, and $W_2^{[i_c]}$, which implies that these gains are not directly obtained by using the linear matrix inequality technique. To solve this difficulty, the method given in [27] is adopted and shown in Fig. 2.

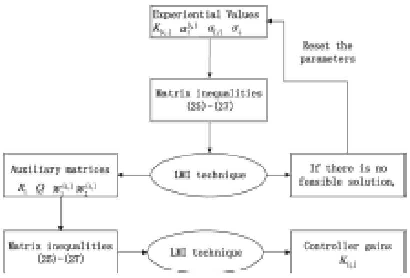


Fig. 2. Block diagram of the controller gain design.

IV. Numerical Example

In what follows, the obtained results are used to solve the problem of sampled-data-based temperature control of a catalytic rod. Similar to [20], a thin and long catalytic rod in a furnace is considered, the spatiotemporal temperature evolution of catalytic rod is governed by a parabolic partial differential equation as follows:

$$\frac{\partial w(z,t)}{\partial t} = \frac{\partial^2 w(z,t)}{\partial z^2} + \beta_T\left(e^{\frac{-\gamma}{1+w(z,t)}} - e^{-\gamma}\right) + \beta_U(b_1(z)u_1(t) + b_2(z)u_2(t) - w(z,t)) \tag{41}$$

with the following boundary conditions:

$$w(0,t) = 0,\ w(\pi,t) = 0$$

where w is the temperature in the reactor, β_T is the heat of reaction, and β_U is the heat transfer coefficient. u_1 and u_2 are the manipulated inputs. Here, we consider the following values of the process parameters:

$$\beta_T = 50,\ \beta_U = 2,\ \gamma = 4$$

$$b_1(z) = \sqrt{\frac{2}{\pi}}\sin(z),\ b_2(z) = \sqrt{\frac{2}{\pi}}\cos(z).$$

Applying the Galerkin method into system (41), a two-dimensions ODE model is derived as follows:

$$\dot{x}(t) = Ax(t) + f(x(t)) + Bu(t) \tag{42}$$

$$x(t) = \left[x_1(t)\ \ x_2(t)\right]^{\mathrm{T}}$$

$$f(x(t)) = \left[f_1(x(t))\ f_2(x(t))\right]^{\mathrm{T}}$$

$$f_1(x(t)) = -\beta_U x_1(t) + \bar{f}_1(t)$$

$$\bar{f}_1(t) = \beta_T\int_0^{\pi}\phi_1(z)\left(e^{\frac{-\gamma}{1+x_1(t)\phi_1(z)+x_2(t)\phi_2(z)}} - e^{-\gamma}\right)dz$$

$$f_2(x(t)) = -\beta_U x_2(t) + \bar{f}_2(t)$$

$$\bar{f}_2(t) = \beta_T\int_0^{\pi}\phi_2(z)\left(e^{\frac{-\gamma}{1+x_1(t)\phi_1(z)+x_2(t)\phi_2(z)}} - e^{-\gamma}\right)dz$$

$$\phi_1 = \sqrt{\frac{2}{\pi}}\sin(z),\ \phi_1 = \sqrt{\frac{2}{\pi}}\sin(2z)$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix},\ B = \begin{bmatrix} \beta_U & 0 \\ 0 & \frac{8\beta_U}{3\pi} \end{bmatrix}.$$

A. NN-Based Approximation Model

To approximate $f(x(t))$, an NN (3), including 16 hidden neurons, is used with $q_i = 1$ and $d_i = 0.5$. The offline training approach is used to obtain the connection weights by BP procedure, where the values of x_1 are taken from $[-0.3533, 20.1467]$ at a step length of 0.5, and the values of x_2 are taken from $[-0.9, 0.9]$ at a step length of 0.1. After 2006 epochs with a typical error 10^{-8}, the upper bound of the approximation errors given in (5) is obtained $\delta = 1.7091$, the weight matrices M_1^* and $(M_2^*)^{\mathrm{T}}$ are, respectively, obtained as follows:

$$\begin{bmatrix} -3.7786 & 0.4564 \\ 0.3962 & 0.0217 \\ 0.8566 & -0.1062 \\ -1.3327 & 0.1627 \\ 5.4544 & -0.5729 \\ -0.7134 & -0.1173 \\ -2.5825 & 0.0010 \\ -1.8118 & -0.0080 \\ -1.5137 & -0.0139 \\ 1.5477 & 0.2251 \\ -5.0764 & 0.5389 \\ -2.4330 & -0.5777 \\ -7.7209 & -0.6448 \\ -8.8179 & 0.8336 \\ -4.9027 & 0.0196 \\ 7.1996 & 0.6463 \end{bmatrix}, \quad \begin{bmatrix} 0.0086 & -0.0366 \\ -2.2985 & -4.1300 \\ -0.3442 & 1.3880 \\ -0.9374 & 4.1924 \\ -0.8907 & 3.0755 \\ -0.3266 & -1.8385 \\ -1.5607 & 0.1936 \\ 4.8347 & -0.8320 \\ -0.9508 & -0.3581 \\ 1.8078 & 7.8632 \\ -0.4219 & 1.4683 \\ 0.2368 & 1.5073 \\ -0.4103 & -1.2150 \\ -0.0765 & 0.2354 \\ -0.5211 & 0.1299 \\ -0.8224 & -2.6087 \end{bmatrix}.$$

B. Scheduling Control Scheme

In this application, there are three controllers to be stochastically scheduled, where stochastic scheduling signal $\ell(t_k)$ is with a finite jump mode set $\mathcal{L} = \{1, 2, 3\}$ and probability distribution $\mathcal{P} = [p_1 = 0.15\ p_2 = 0.36\ p_3 = 0.49]$. The switching signal $r(t_k)$ of the execution time allotted by the scheduler is with finite switching mode sets ${}^{[1]}\mathcal{R} = \{1.1\}$, ${}^{[2]}\mathcal{R} = \{2.1, 2.8\}$, ${}^{[3]}\mathcal{R} = \{3.1, 3.8\}$, and associated probability distributions

$$\begin{aligned} {}^{[1]}\wp &= [\sigma_{1;1.1} = 1] \\ {}^{[2]}\wp &= [\sigma_{2;2.1} = 0.45\ \sigma_{2;2.8} = 0.55] \\ {}^{[3]}\wp &= [\sigma_{3;3.1} = 0.54\ \sigma_{3;2.8} = 0.46]. \end{aligned}$$

Here, we take $T_c = t_{k+1} - t_k = 10h$ and $h = 0.0002$, The execution intervals allotted by the scheduler are

$$\begin{aligned} &{}^{[1]}\Gamma_1 : [t_k, t_k + 10h) \\ &{}^{[2]}\Gamma_1 : [t_k, t_k + 4h),\ [t_k + 4h, t_k + 6h) \\ &{}^{[2]}\Gamma_2 : [t_k, t_k + 8h),\ [t_k + 8h, t_k + 10h) \\ &{}^{[3]}\Gamma_1 : [t_k, t_k + 3h),\ [t_k + 3h, t_k + 8h),\ [t_k + 8h, t_k + 10h) \\ &{}^{[3]}\Gamma_2 : [t_k, t_k + 5h),\ [t_k + 5h, t_k + 7h), [t_k + 7h, t_k + 10h). \end{aligned}$$

With taking the following parameters:

$$R_1 = \begin{bmatrix} 0.1098 & -0.0001 \\ -0.0001 & 0.1763 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.0055 & -0.0000 \\ -0.0000 & 0.0089 \end{bmatrix}$$

$$W_1^{[1]} = \begin{bmatrix} 0.0499 & -0.0001 \\ -0.0001 & 0.0012 \end{bmatrix}, \quad W_1^{[2]} = \begin{bmatrix} -0.0132 & -0.0004 \\ -0.0004 & -0.0062 \end{bmatrix}$$

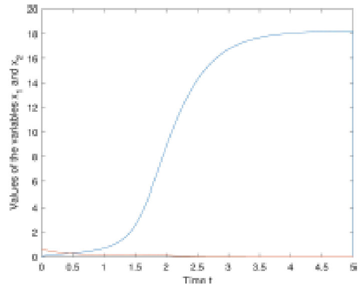


Fig. 3. Trajectories of open-loop system (42).

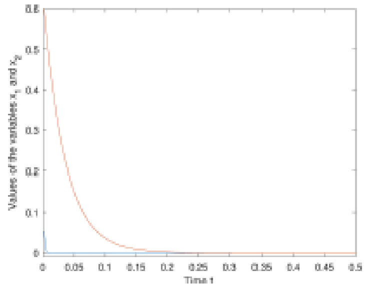


Fig. 4. Trajectories of closed-loop system under the proposed stochastic scheduling control.

$$W_1^{[3]} = \begin{bmatrix} 0.1231 & 0.0001 \\ 0.0001 & -0.0326 \end{bmatrix}, \quad W_2^{[1]} = \begin{bmatrix} 0.0005 & 0.0000 \\ 0.0000 & 0.0096 \end{bmatrix}$$

$$W_2^{[2]} = \begin{bmatrix} 0.0002 & -0.0001 \\ -0.0001 & 0.0062 \end{bmatrix}, \quad W_2^{[3]} = \begin{bmatrix} 0.0010 & 0.0000 \\ 0.0000 & 0.0108 \end{bmatrix}$$

it is derived by using Theorem 2 that

$$\begin{aligned} K_{[1]} &= \begin{bmatrix} -178.5438 & -1.0681 \\ -1.0681 & -15.0355 \end{bmatrix} \\ K_{[2]} &= \begin{bmatrix} -193.7567 & -2.3918 \\ -2.3918 & -20.5714 \end{bmatrix} \\ K_{[3]} &= \begin{bmatrix} -137.1727 & -0.9542 \\ -0.9542 & -12.3555 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\alpha_\ell^{[1]}\alpha_{[1]} = -91.8977,\ \alpha_\ell^{[2]}\alpha_{[1]} = 2.8727 \\ &\alpha_\ell^{[2]}\alpha_{[2]} = -132.4239,\ \alpha_\ell^{[3]}\alpha_{[1]} = 7.2613 \\ &\alpha_\ell^{[3]}\alpha_{[2]} = -85.8243,\ \alpha_\ell^{[3]}\alpha_{[3]} = -36.1840. \end{aligned}$$

Simulations: Take the initial condition of the catalytic rod system (41) as follows:

$$w(z, 0) = 0.05\sqrt{\frac{2}{\pi}}\sin(z) + 0.6\sqrt{\frac{2}{\pi}}\sin(2z).$$

Applying Galerkin's approach into the initial condition, the initial value of ODE model (42) is obtained as $x(0) = [0.05\ 0.6]^{\mathrm{T}}$. Using the obtained controller gains and scheduling scheme, state trajectories of open-loop system (42) are

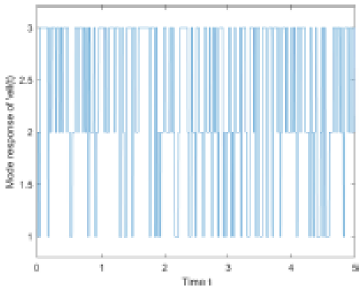


Fig. 5. Response of Markov process $\ell(t)$.

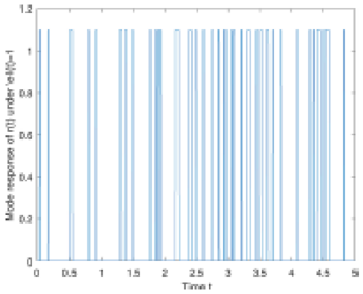


Fig. 6. Response of stochastic mode $r(t) \in {}^{[1]}\mathcal{R}$.

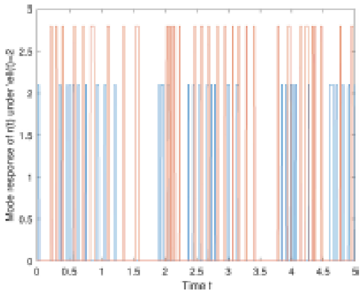


Fig. 7. Response of stochastic mode $r(t) \in {}^{[2]}\mathcal{R}$.

shown in Fig. 3. It is clear that open-loop system (42) is unstable at $x(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$. Using the proposed control method in this article, trajectories of the controlled system are given in Fig. 4. In view of this figure, two trajectories converge to the zero state after time $t = 0.3$, which implies that the scheduling scheme can guarantee a.s. stability of the system. The response of Markov mode $\ell(t) \in \{1, 2, 3\}$ is shown in Fig. 5. The response of stochastic mode $r(t) \in {}^{[1]}\mathcal{R}$ is shown in Fig. 6 under Markov mode $\ell(t) = 1$. The response of stochastic mode $r(t) \in {}^{[2]}\mathcal{R}$ is shown in Fig. 7 under Markov mode $\ell(t) = 2$.

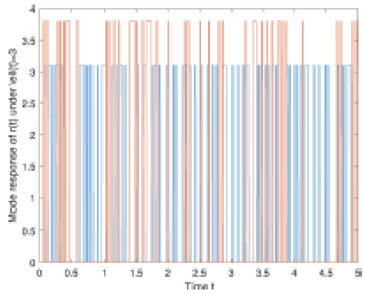


Fig. 8. Response of stochastic mode $r(t) \in {}^{[3]}\mathcal{R}$.

The response of stochastic mode $r(t) \in {}^{[3]}\mathcal{R}$ is shown in Fig. 8 under Markov mode $\ell(t) = 3$.

V. Conclusion

The problem of stability in almost sure sense of UNSs with a stochastic scheduling scheme is studied. Since numerous simulation studies and actual industrial implementations show that the neural network is a good candidate to handle the design difficulty resulted by unknown nonlinearities. In the stochastic scheduling scheme, both the choice of controllers and their execution time allotted the scheduler are random. General sufficient conditions based on the probability distribution of almost sure Lyapunov stability are obtained by using some stochastic techniques. It is worth mentioning that the phenomenon of packet disordering often occurs for the networked system when the data packet is over the communication network. Inspired by this, in future work, the problem of anytime scheduling control is going to study under packet disordering.

References

[1] L. Palopoli, L. Abeni, G. Bolognini, B. Allotta, and F. Conticelli, "Novel scheduling policies in real-time multithread control system design," *Control Eng. Pract.*, vol. 10, no. 10, pp. 1095–1110, Oct. 2002.

[2] G. Bernat, A. Burns, and A. Llamosi, "Weakly hard real-time systems," *IEEE Trans. Comput.*, vol. 50, no. 4, pp. 308–321, Apr. 2001.

[3] D. Liu, X. S. Hu, M. D. Lemmon, and Q. Ling, "Firm real-time system scheduling based on a novel QoS constraint," *IEEE Trans. Comput.*, vol. 55, no. 3, pp. 320–333, Mar. 2006.

[4] L. Greco, D. Fontanelli, and A. Bicchi, "Design and stability analysis for anytime control via stochastic scheduling," *IEEE Trans. Autom. Control*, vol. 56, no. 3, pp. 571–585, Mar. 2011.

[5] G. Wang and Y. Sun, "Almost sure stabilization of continuous-time jump linear systems via a stochastic scheduled controller," *IEEE Trans. Cybern.*, vol. 52, no. 5, pp. 2712–2724, May 2022, doi: 10.1109/TCYB.2020.3021424.

[6] H. Zhang, J. Liu, and S. Xu, "Practical stability and event-triggered load frequency control of networked power systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 10, pp. 6296–6304, Oct. 2022.

[7] X. Xie, D. Yue, J. H. Park, and J. Liu, "Enhanced stabilization of discrete-time Takagi-Sugeno fuzzy systems based on a comprehensive real-time scheduling model," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 2, pp. 881–892, Feb. 2022.

[8] X. Xie, Q. Zhou, D. Yue, and H. Li, "Relaxed control design of discrete-time takagi-sugeno fuzzy systems: An event-triggered real-time scheduling approach," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2251–2262, Dec. 2018.

[9] S. Hu, Z. Cheng, D. Yue, C. Dou, and Y. Xue, "Bandwidth allocation-based switched dynamic triggering control against DoS attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 10, pp. 6050–6061, Oct. 2021.

[10] C. Hu, H. He, and H. Jiang, "Fixed/preassigned-time synchronization of complex networks via improving fixed-time stability," *IEEE Trans. Cybern.*, vol. 51, no. 6, pp. 2882–2892, Jun. 2021.

[11] C. Hu and H. Jiang, "Special functions-based fixed-time estimation and stabilization for dynamic systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 5, pp. 3251–3262, May 2022.

[12] K. S. Narendra, "Neural networks for control theory and practice," *Proc. IEEE*, vol. 84, no. 10, pp. 1385–1406, Oct. 1996.

[13] J. A. K. Suykens, J. Vandewalle, and B. De Moor, *Artificial Neural Networks for Modeling and Control of Non-Linear Systems*. Norwell, MA, USA: Kluwer, 1996.

[14] Y. Xie and Q. Ma, "Adaptive event-triggered neural network control for switching nonlinear systems with time delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 2, pp. 729–738, Feb. 2023, doi: 10.1109/TNNLS.2021.3100533.

[15] Q. Meng and Q. Ma, "Global stabilization for a class of stochastic nonlinear time-delay systems with unknown measurement drifts and its application," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 12, pp. 7153–7160, Dec. 2022, doi: 10.1109/TNNLS.2021.3084295.

[16] H. Shen, T. Wang, J. Cao, G. Lu, Y. Song, and T. Huang, "Nonfragile dissipative synchronization for Markovian memristive neural networks: A gain-scheduled control scheme," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 6, pp. 1841–1853, Jun. 2019, doi: 10.1109/TNNLS.2018.2874035.

[17] H. Shen, Y. Zhu, L. Zhang, and J. H. Park, "Extended dissipative state estimation for Markov jump neural networks with unreliable links," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 346–358, Feb. 2017.

[18] J. Ma, S. Xu, Q. Ma, and Z. Zhang, "Event-triggered adaptive neural network control for nonstrict-feedback nonlinear time-delay systems with unknown control directions," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 10, pp. 4196–4205, Oct. 2020, doi: 10.1109/TNNLS.2019.2952709.

[19] J. Ma, S. Xu, Y. Li, Y. Chu, and Z. Zhang, "Neural networks-based adaptive output feedback control for a class of uncertain nonlinear systems with input delay and disturbances," *J. Franklin Instit.*, vol. 355, no. 13, pp. 5503–5519, 2018.

[20] H. N. Wu and H. X. Li, "A Galerkin/neural-network-based design of guaranteed cost control for nonlinear distributed parameter systems," *IEEE Trans. Neural Netw.*, vol. 19, no. 5, pp. 795–807, May 2008.

[21] K. I. Funahashi, "On the approximate realization of continuous mappings by neural networks," *Neural Netw.*, vol. 2, no. 3, pp. 183–192, 1989.

[22] X. R. Mao, *Stochatic Differential Equations and Applications*. 2nd ed. Chichester, U.K.: Horwood Publ., 2007.

[23] J. Chen, S. Xu, and B. Zhang, "Single/multiple integral inequalities with applications to stability analysis of time-delay systems," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3488–3493, Jul. 2017.

[24] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[25] J. Chen, S. Xu, W. Chen, B. Zhang, Q. Ma, and Y. Zou, "Two general integral inequalities and their applications to stability analysis for systems with time-varying delay," *Inter. J. Robust Nonlinear Control*, vol. 26, no. 18, pp. 4088–4103, 2016.

[26] K. Liu, V. Suplin, and E. Fridman, "Stability of linear systems with general sawtooth delay," *IMA J. Math. Control Inf.*, vol. 27, no. 4, pp. 419–436, Dec. 2010.

[27] H. Zhang, J. Liu, and S. Xu, "H-infinity load frequency control of networked power systems via an event-triggered scheme," *IEEE Trans. Ind. Electron.*, vol. 67, no. 8, pp. 7104–7113, Aug. 2020.

Huihui Ji received the B.S. degree in mathematics and applied mathematics from Xinyang Normal University, Xinyang, China, in 2013, the M.S. degree in applied mathematics from the Chongqing University of Technology, Chongqing, China, in 2016, and the Ph.D. degree in control science and engineering from Jiangnan University, Wuxi, China, in 2020.

From 2018 to 2019, she was a joint supervisory Ph.D. student with the University of Waterloo, Waterloo, ON, Canada. Since July 2020, she has been a Lecturer with the School of Mathematics, Nanjing Audit University, Nanjing, China. Her current research interests include networked control systems, stochastic systems, and distributed parameter systems.

Shengyuan Xu (Senior Member, IEEE) received the B.Sc. degree in mathematics from Hangzhou Normal University, Hangzhou, China, in 1990, the M.Sc. degree in operations research and cybernetics from Qufu Normal University, Qufu, China, in 1996, and the Ph.D. degree in control theory from the Nanjing University of Science and Technology, Nanjing, China, in 1999.

From 1999 to 2000, he was a Research Associate with the Department of Mechanical Engineering, University of Hong Kong, Hong Kong. From December 2000 to November 2001 and from December 2001 to September 2002, he was a Postdoctoral Researcher with CESAME, Universite catholique de Louvain, Ottignies-Louvain-la-Neuve, Belgium, and the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada, respectively. Since November 2002, he has been a Professor with the School of Automation, Nanjing University of Science and Technology. His current research interests include robust filtering and control, singular systems, time-delay systems, neural networks, multidimensional systems, and nonlinear systems.

Dr. Xu was a recipient of the National Excellent Doctoral Dissertation Award in 2002 from the Ministry of Education of China. He obtained a grant from the National Science Foundation for Distinguished Young Scholars of P. R. China in 2006. He was awarded a Cheung Kong Professorship in 2008 from the Ministry of Education of China. He is a member of the editorial boards of the *Transactions of the Institute of Measurement and Control* and the *Journal of the Franklin Institute*.

He Zhang received the B.S. degree in statistics and the M.S. degree in operational research and cybernetics from the School of Science, Chongqing University of Technology, Chongqing, China, in 2012 and 2015, respectively, and the Ph.D. degree in control science and engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2021.

From 2015 to 2017, he was a Lecturer with Baise University, Baise, China. From 2018 to 2019, he was a joint supervisory Ph.D. student with the University of Waterloo, Waterloo, ON, Canada. Since July 2021, he has been a Lecturer with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing. His current research interests include networked control systems, stochastic systems, distributed parameter systems, and development of advanced control algorithm for chemical process automation.

南京理工大学图书馆

检 索 报 告

委托人： 季慧慧

委托人单位： 南京审计大学

检索要求： 季慧慧发表的论文：1、被 SCIE 数据库收录情况；
2、论文发表期刊的分区情况。

检索数据库： SCI EXPANDED（科学引文索引扩展版 1981-2024）
中科院文献情报中心期刊分区表升级版（2023 年）

检索日期： 2024 年 03 月 29 日

检 索 结 果

本次检索根据委托人 季慧慧 所提供的论文目录及其检索要求，通过对以上数据库进行检索，检索结果如下：

1. 收录情况

季慧慧以第一作者发表的论文被 SCIE 数据库收录 1 篇。

2. 论文发表期刊的分区情况

以上 SCIE 论文的发表期刊 *IEEE TRANSACTIONS ON SYSTEMS MAN CYBERNETICS-SYSTEMS* 在中科院文献情报中心期刊分区表升级版（2023）的计算机科学（大类）学科中为 1 区 Top 期刊。

检索结果详见附件（共计 2 页）。

特此证明！

检索证明人：于丽娜 审核人：周

教育部科技查新工作站（L28）
南京理工大学图书馆
2024 年 03 月 29 日

210094 南京市孝陵卫 200 号 Tel：025-84314962 Email:tsgzx6@njust.edu.cn

IEEE Transactions on Systems Man Cybernetics-Systems

刊名	IEEE Transactions on Systems Man Cybernetics-Systems
年份	2023
ISSN	2168-2216
Review	否
Open Access	否
Web of Science	SCIE

	学科	分区	Top期刊
大类	计算机科学	1	是
小类	AUTOMATION & CONTROL SYSTEMS 自动化与控制系统	1	-
	COMPUTER SCIENCE, CYBERNETICS 计算机：控制论	1	

Clarivate

Web of Science™

1 record(s) printed from Clarivate Web of Science

第 1 条，共 1 条

标题: Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems

作者: Ji, HH (Ji, Huihui); Xu, SY (Xu, Shengyuan); Zhang, H (Zhang, He)

来源出版物: IEEE TRANSACTIONS ON SYSTEMS MAN CYBERNETICS-SYSTEMS **卷:** 54 **期:** 1 **页:** 97-106 **DOI:** 10.1109/TSMC.2023.3305518 **Early Access Date:** SEP 2023 **Published Date:** 2024 JAN

Web of Science 核心合集中的 "被引频次": 0

被引频次合计: 0

入藏号: WOS:001064625700001

语言: English

文献类型: Article

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Math, Nanjing 211815, Peoples R China.
[Xu, Shengyuan] Nanjing Univ Sci & Technol, Sch Automat, Nanjing 210094, Peoples R China.
[Zhang, He] Nanjing Tech Univ, Coll Elect Engn & Control Sci, Nanjing 211800, Peoples R China.

通讯作者地址: Xu, SY (通讯作者)，Nanjing Univ Sci & Technol, Sch Automat, Nanjing 210094, Peoples R China.

电子邮件地址: jihuihui2020@nau.edu.cn; syxu@njust.edu.cn; zhanghe@njtech.edu.cn

Affiliations: Nanjing Audit University; Nanjing University of Science & Technology; Nanjing Tech University

ISSN: 2168-2216

eISSN: 2168-2232

输出日期: 2024-03-29

End of File

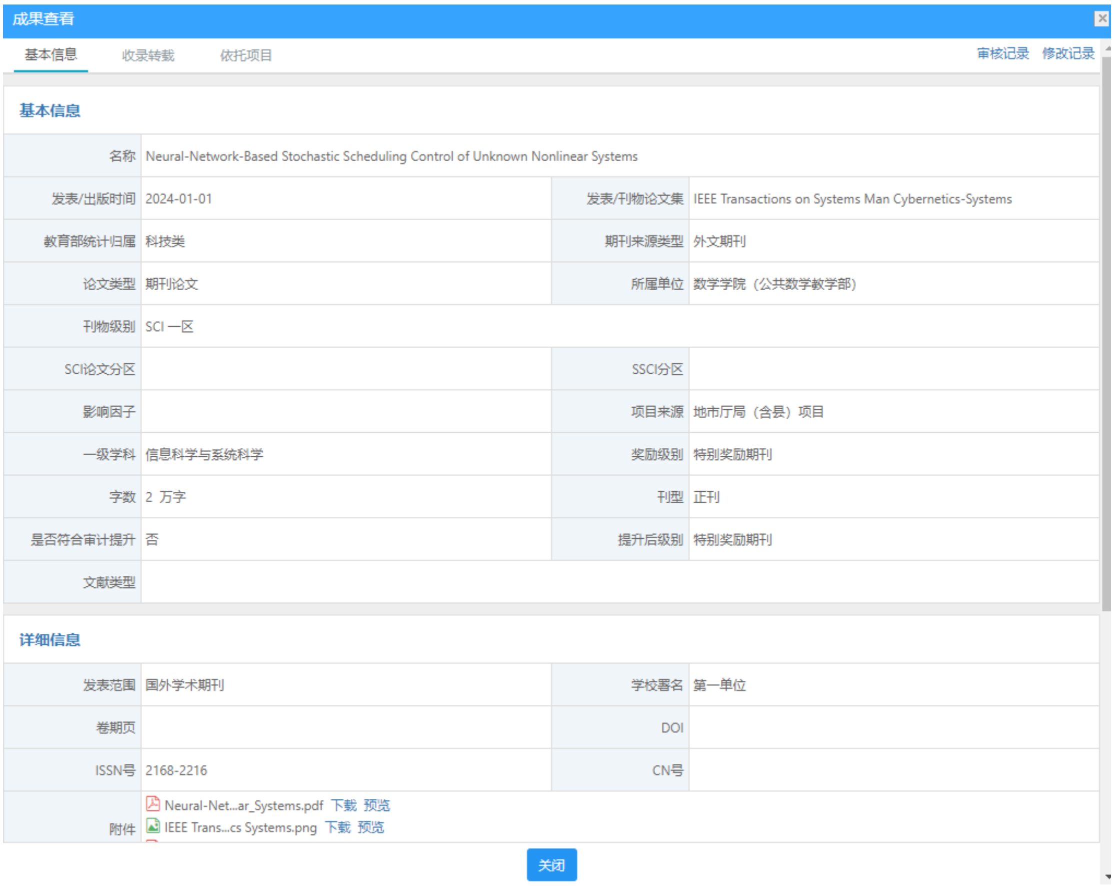
Clarivate

科研项目 高教项目 科研成果 学术活动 高被引 校级项目申报 纵向项目申报 教研项目申报 入账办理 更多 季慧慧

成果列表 考核时间更新 新增 待确认网推论文

名称 模糊 审核状态 所属单位 第一作者姓名 模糊 更多 查询

名称	所有作者	成果日期	成果类型	审核状态	操作
Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems	季慧慧,徐胜元（外）,张贺（外）	2024-01-01	论文	学校通过	收录补录 转载补录
Networked sampled-data control of distributed parameter systems via distributed sensor networks SCIE	季慧慧,Cui, Baotong（外）,Liu, Xinzhi（外）	2021-07-01	论文	学校通过	收录补录 转载补录
Event-triggered predictor-based control of distributed parameter systems	季慧慧,崔宝同（外）,刘新芝（外）	2021-03-01	论文	学校通过	收录补录 转载补录
2023年度江苏省自动化学会科学技术奖	张贺（外）,喻果（外）,董书华（外）,季慧慧,彭岳（外）,蒲翠梅（外）	2024-02-23	获奖	学校通过	
Finite-time quasi- synchronization of stochastic multilayer networks with energy consumption estimation	徐玉华,Wu Xiaoqun（外）,Wan Xiaoxiao（外）,季慧慧,周世超	2022-04-16	论文	学校通过	
Event-generator-based H∞ control of fuzzy distributed parameter systems	季慧慧,崔宝同（外）	2022-03-28	论文	学校通过	收录补录 转载补录
概率论与数理统计课程思政的探索与思考	季慧慧	2022-01-05	论文	学校通过	收录补录 转载补录



成果查看

基本信息 收录转载 依托项目 审核记录 修改记录

基本信息

名称	Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems		
发表/出版时间	2024-01-01	发表/刊物论文集	IEEE Transactions on Systems Man Cybernetics-Systems
教育部统计归属	科技类	期刊来源类型	外文期刊
论文类型	期刊论文	所属单位	数学学院（公共数学教学部）
刊物级别	SCI 一区		
SCI论文分区		SSCI分区	
影响因子		项目来源	地市厅局（含县）项目
一级学科	信息科学与系统科学	奖励级别	特别奖励期刊
字数	2 万字	刊型	正刊
是否符合审计提升	否	提升后级别	特别奖励期刊
文献类型			

详细信息

发表范围	国外学术期刊	学校署名	第一单位
卷期页		DOI	
ISSN号	2168-2216	CN号	
附件	Neural-Net...ar_Systems.pdf 下载 预览 IEEE Trans...cs Systems.png 下载 预览		

关闭

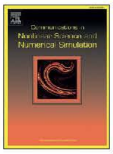
2. Networked sampled-data control of distributed parameter systems via distributed sensor networks (中科院 1 区 TOP 期刊、特别奖励期刊) (1/3)

Commun Nonlinear Sci Numer Simulat 98 (2021) 105773

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns



Research paper

Networked sampled-data control of distributed parameter systems via distributed sensor networks

Huihui Ji[a,*], Baotong Cui[b,c], Xinzhi Liu[d]

[a] School of Statistics and Mathematics, Nanjing Audit University, 86 West Yushan Road, Nanjing 211815, China
[b] Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, China
[c] School of IoT Engineering, Jiangnan University, Wuxi 214122, China
[d] Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

ARTICLE INFO

Article history:
Received 13 September 2020
Revised 9 February 2021
Accepted 17 February 2021
Available online 19 February 2021

Keywords:
Distributed parameter systems
Sampled-data control
Sensor network
Time delay
Exponential stability and stabilization

ABSTRACT

This paper studies the problem of the distributed networked sampled-data control for a class of distributed parameter systems with spatially-dependent diffusion term. In view of limited fixed sampling spatial points, the distributed sensor network is proposed to obtain the sampled-data measurements of the state which can efficaciously circumvent blind sampling area or sampling error. A distributed sampled data output feedback controller based on the distributed sensor network is designed to ensure the stabilization of the distributed parameter systems with the time delays induced by the communication network. Based on Lyapunov method, linear matrix inequality technique and time-delay approach, the global exponential stability criteria are obtained for the closed-loop distributed parameter systems under three different boundary conditions, respectively. Finally, the numerical simulation proves the effectiveness of the controller, and numerical comparison shows that the proposed control method is less conservative.

1. Introduction

With the advanced development of the digital sensor in the last few decades, sampled-data control has attracted special attention of numerous researchers due to its advantages, such as high accuracy and reliability, effective interference suppression, and good versatility. The design of sampled-data control scheme plays the crucial role in the applications of digital implementation issues as well as theoretical development. Consequently, a great deal of studies have been focused on the sampled-data control problem for various kinds of finite dimensional systems such as the works [1–4].

Notice the fact that a large body of real systems including heat transfer process and reaction-diffusion process cannot be described by lumped parameter systems with finite dimensions, an increasing interest is given to the sampled-data control of distributed parameter systems (DPSs) with infinite dimensions. The study of sampled-data control problems of DPSs is becoming active and has become a hot spot in the control field. Recently, Tarn et al. proposed a stabilization scheme in [5] by using periodic output feedback control instead of continuously monitoring. Based on a generalized sampled method with a weighting function, Logemann et al. in [6] derived the exponential stability of the infinite-dimensional

* Corresponding author.
E-mail addresses: jihuihui2020@nau.edu.cn (H. Ji), btcui@jiangnan.edu.cn (B. Cui), xzliu@uwaterloo.ca (X. Liu).

https://doi.org/10.1016/j.cnsns.2021.105773

system. In [7], Logemann et al. constructed a sampled-data feedback controller to stabilize the infinite-dimensional system and a series of exponentially stable conditions were presented by using semigroup theory. By employing piecewise polynomial controls, Rebarber and Townley in [8] obtained some necessary and sufficient conditions for stabilization of continuous-time DPSs. Moreover, Logemann in [9] presented the necessary and sufficient condition for the existence of the stabilizing linear sampled-data controller. Recently, Cheng et al. in [10] proposed a sampled-data strategy for a boundary control problem of a class of DPSs. These sampled controllers seem to be easy to implement in that the controlled objects are all linear time-invariant systems. For nonlinear infinite-dimensional systems, Wang and Wu in [11] studied the sampled-data fuzzy control problem. Moreover, for a class of nonlinear DPSs with sampled-data measurements, Ghantasala and El-Farra in [12] considered the active fault-tolerant control issue, where the model reduction method (e.g., Galerkin's technique) was suggested to obtain a finite-dimensional system which can represent the main characteristics of the studied DPSs, and the finite-dimensional controller was designed based on the finite-dimensional system. However, it seems to be a huge challenge to achieve an exact performance of the original DPSs because of the truncation before controller design.

To avoid this drawback, the design proposals of infinite-dimensional controllers were proposed recently to obtain better control performance, see the works [13–17] and the references therein. Selivanov and Fridman in [13] used the time-delay approach to study the sampled-data relay control problem for semilinear parabolic system. Kang and Fridman in [14] employed the time-delay approach and Lyapunov-Krasovskii method to deal with the sampled-data control problem and to derive the regional exponential stability conditions. Most recently, Wang et al. in [15,16] considered to combine Lyapunov-Krasovskii method and Takagi-Sugeno fuzzy model approach to derive the sampled-data exponential stability conditions. However, the results in [13] were of regional or semi-global stabilization, and the approach in [14,15] was inapplicable to the DPSs with spatially-dependent diffusion coefficients. To solve the global stability problems for DPSs with spatially-dependent diffusion coefficients and uncertain nonlinear terms, Fridman and Blighovsky in [17] presented a robust sampled-data controller and derived the decay rate of the exponential convergence. However, the problem of allocating resources in a distributed fashion is ignored in the research [17], where the sensors collect information just from a single resource, which may poses the blind sampling area or sampling error. Then, to make full use of sensor network resources effectively is an advantageous approach to obtain the precise sampled data of the systems.

Motivated by the above discussion, in this paper, we study the network-based sampled-data control of a class of DPSs with spatially-dependent diffusion term. In view of limited fixed sampling spatial points, the distributed sensor network is proposed to obtain the sampled-data measurements of the state which is consisting of groups of sensors is considered in the sampling processing, and the sensors provide a series of state measurements to their own sensor group by the interaction among sensor nodes based on the prescribed sensing topology, which can efficaciously circumvent blind sampling area or sampling error. Then, a networked sampled-data static output feedback controller is proposed to stable DPSs with the time delays induced by the communication network. The global exponential stability criteria in terms of LMIs are obtained for DPSs with three different boundary conditions, respectively. Further, the upper bounds on the sampling intervals and the upper bounds on the resulting decay rate are obtained in this paper. Finally, a numerical example under three different boundary conditions is given to illustrate the effectiveness of the obtained results.

The main contribution and advantage of this paper can be summarized as follows:

(1) A distributed sensor networked is employed to obtain more precise sampled data of the systems, which can efficaciously circumvent blind sampling area or sampling error caused by a single sensor, where M sensors are considered to provide the measurement for spatial sampled point in each sub-domains by collaborative interaction information among sensors according to a prescribed sensing topology, which yields improved environment perception and system efficiency while providing desired information;
(2) Different from the finite-dimensional sampled data controllers in [12], the static output feedback controller proposed in this paper is an infinite-dimensional sampled data controller based on the output measurement of the infinite-dimensional distributed parameter systems without any information lost, which can accurately and effectively stable the DPSs with spatially-dependent diffusion term. Compared with the impulsive controller in [18], which requires a large number of spatial point sampling measurements, the networked sampled-data controller based on a finite number of point output measurements in this paper is easy and convenient to implement;
(3) Compared with the stability criteria in [17,19], the stability criteria in this paper make better control performance of the system and relax the restrictions on the system by using our controller and control method, which can be shown in numerical example.

Notations: In this paper, the set of all real numbers is denoted by $\mathbb{R}$; The set of n-dimensional vectors is denoted by $\mathbb{R}^n$; $\mathbb{R}^{m\times n}$ stands for the set of all real $m \times n$ matrices; I represents the appropriate dimension identity matrix; I_n represents the $n \times n$ identity matrix; $*$ stands for the symmetric matrix; *exp* means exponent; Let $\mathcal{H} = \mathcal{L}_2([0, l]; \mathbb{R})$ be a Sobolev space which is produced by square integrable absolute functions $\mathscr{M}(\vartheta, t) \in \mathbb{R}^n$, $\vartheta \in [0, l] \subset \mathbb{R}$, $\forall t \geq 0$; $\mathscr{C}^1$ stands for a set of smooth functions; $\nabla \mathscr{M}(\vartheta, t)$ stands for the partial derivative of $\mathscr{M}(\vartheta, t)$ with respect to ϑ.

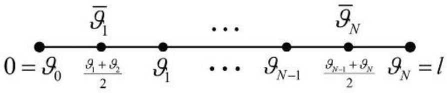


Fig. 1. Sampling spatial points.

2. Problem formulation and preliminary

2.1. Plant: diffusion semilinear DPSs

Consider a spatially-distributed processes model, which dynamic trajectories are governed by the following DPSs

$$\frac{\partial \mathscr{M}(\vartheta,t)}{\partial t} = \Delta D(\vartheta)\mathscr{M}(\vartheta,t) + E\nabla\mathscr{M}(\vartheta,t) + A\mathscr{M}(\vartheta,t) + f(\mathscr{M}(\vartheta,t),t) + u(\vartheta,t), \tag{1}$$

where $\mathscr{M}(\vartheta,t) = [\mathscr{M}_1(\vartheta,t),\ldots,\mathscr{M}_n(\vartheta,t)]^T \in \mathbb{R}^n$ is state variable; $\vartheta \in [0,l], t \geq 0$; the diffusion term is considered as

$$\Delta D(\vartheta)\mathscr{M}(\vartheta,t) = \left[\frac{\partial}{\partial\vartheta}(d_1(\vartheta)\mathscr{M}_\vartheta(\vartheta,t)),\ldots,\frac{\partial}{\partial\vartheta}(d_n(\vartheta)\mathscr{M}_\vartheta(\vartheta,t))\right],$$

is the diffusion term with $d_i(\vartheta) \in \mathscr{C}^1$ such that $0 < d_i^0 < d_i(\vartheta)$ for $\vartheta \in [0,l], i = 1,2,\ldots,n$; $f(\mathscr{M}(\vartheta,t)) = [f_1(\mathscr{M}_1(\vartheta,t)),\ldots,f_n(\mathscr{M}_n(\vartheta,t))]^T$ ($f: \mathbb{R}^n \times [0,+\infty) \to \mathbb{R}^n$) is considered as continuous and differentiable; $u(\vartheta,t) \in \mathbb{R}^n$ is the controlled input; A and E are regarded as known constant matrices with appropriate dimensions.

In this paper, the boundary conditions and initial condition of the system (1) is respectively

1. Dirichlet conditions

$$\mathscr{M}(0,t) = \mathscr{M}(l,t) = 0, \quad t \in [0,+\infty). \tag{2}$$

2. Neumann conditions

$$\nabla\mathscr{M}(0,t) = \nabla\mathscr{M}(l,t) = 0, \quad t \in [0,+\infty). \tag{3}$$

3. and mixed boundary conditions

$$\nabla\mathscr{M}(0,t) = B\mathscr{M}(0,t), \quad \mathscr{M}(l,t) = 0, \quad t \in [0,+\infty), \tag{4}$$

and the initial condition is

$$\mathscr{M}(\vartheta,0) = \mathscr{M}_0(\vartheta), \quad \vartheta \in [0,l], \tag{5}$$

where $B = diag\{b_1, b_2, \ldots, b_n\}$ is a nonnegative diagonal matrix, and constants $b_i < 0$, $i = 1,2,\ldots,n$.

2.2. Networked sampled-data communication scheme

To facilitate sampling for the DPSs, we divide the interval [0, l] into N subintervals by the points $0 = \vartheta_0 < \vartheta_1 < \ldots < \vartheta_N = l$, where $\Omega_i = [\vartheta_{i-1}, \vartheta_i)$, $0 < \vartheta_i - \vartheta_{i-1}$, $i \in \aleph = \{1,\ldots,N\}$. The middle of each subinterval is denoted by $\bar{\vartheta}_i = \frac{\vartheta_i+\vartheta_{i-1}}{2}$, $i \in \aleph$, which is shown in Fig. 1.

For the sampling points $\bar{\vartheta}_1, \bar{\vartheta}_2, \ldots, \bar{\vartheta}_N$, we adopt synchronized sampling strategy with a series of sampling instants

$$0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots,$$

with $\lim_{k\to\infty} t_k = \infty$.

There are N groups of sensors in the interval [0, l] to provide the sampling data. More specifically, a group of sensors consisting of M sensors are distributed in each subintervals $\Omega_i, i \in \aleph$. The interconnection between the group of sensors in each subinterval $\Omega_i, i \in \aleph$ can be represented by the following the directed weighted graph $G = \{U, V, W\}$, where $U = \{U_1, U_2, \ldots, U_M\}$ is the index of the sensor node, $V \subseteq U \times U$ is a set of edges between pairs of sensors, the edges $V_{ms} = (U_m, U_s)$ in G means that the sensor U_m, $m \in \nu = \{1,2,\ldots,M\}$ can receive information from the node U_s, $s \in \nu = \{1,2,\ldots,M\}$. The weighted adjacency matrix of edges is $W = \{\varepsilon_{ms}\}_{m,s=1}^M$, where $\varepsilon_{mm} = 0$, $\varepsilon_{ms} > 0$ equals to $V_{ms} \in V$. The degree matrix of the graph is $\mathcal{V} = diag\{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_M\}$, $\mathcal{V}_m = \sum_{s=1}^M \varepsilon_{ms}$, $m \in \nu = \{1,2,\ldots,M\}$. The Laplace matrix of the graph is $\mathcal{D} = \mathcal{V} - W$. The flow diagram of sampled-data collection surrounding the sensor node m is depicted in Fig. 2, where the sensor node m provides the measurements based on the interaction with the information among itself and its neighboring nodes dispersedly deployed in the sensor field.

In view of Fig. 2, for each subinterval $\Omega_i, i \in \aleph$, we can get M measurements for the ith output measurement $y_{i_m}(x,t_k)$ at each sampling instants t_k, that is,

$$y_{i_m}(\bar{\vartheta}_i,t_k) = C_{i_m}\mathscr{M}(\bar{\vartheta}_i,t_k), \quad i \in \aleph, \quad m \in \nu, \quad k = 0,1,2,\ldots.$$

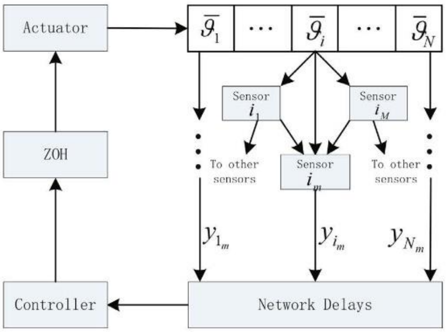


Fig. 2. System (1) with networked sampled-data communication scheme.

where $C_{i_m}, i \in \aleph, m \in \nu$ is a constant matrix with appropriate dimension.

And the aggregated data is processed and presented as

$$\bar{y}_{i_m}(\bar{\vartheta}_i, t) = \sum_{s=1}^{M} \varepsilon_{ms} y_{i_m}(\bar{\vartheta}_i, t), \tag{6}$$

In practical, the induced network delays may be different for each sampling data transmitted in the communication network. In this paper, we consider a set of time-delays caused by the communication network for the system (1). Without losses of generality, we assume that the constant delay for the kth sampling data from the sensor to the controller is denoted by τ_k, and the delay $\overline{\tau} = \sup_k\{\tau_k\}$.

In the following, we assume:

1) every sampling and signal transmission is successful and there is no data packet dropouts.
2) the sampling intervals are bounded

$$0 < h_{\min} \le t_{k+1} - t_k = h_{k+1},$$

whereas each sampling period may not be fixed for $k \in 0,\ 1,\ 2,\ \ldots$. Then, we have

$$t_{k+1} + \tau_{k+1} - (t_k + \tau_k) = h_{k+1} + \tau_{k+1} - \tau_k,$$

where $t_k + \tau_k$ means the updating instant of control inputs, and $t_{k+1} + \tau_{k+1}$ stands for the next updating instant. Then, the maximum time span between two adjacent sampling instants can be denoted by $\tau_M = \max_k\{h_{k+1} - \tau_k + \tau_{k+1}\}$.
3) there is no the Zeno behavior because of the positive lower bound $h_{\min}$ on the sampling time intervals [20].

2.3. Delay-dependent sampled-data feedback control model

A networked sampled-data controller is given:

$$u(\vartheta, t) = \sum_{i=1}^{N} \chi_i(\vartheta) u_i(\vartheta, t), \quad u_i(\vartheta, t) = K_i[\mathcal{D} \otimes I_n] C_i \mathscr{M}(\bar{\vartheta}_i, t_k) \in \mathbb{R}^n, \quad \chi_i(\vartheta) = \begin{cases} 1, & \vartheta \in \Omega_i, \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$
$$K_i = [K_{i_1}, K_{i_2}, \ldots, K_{i_M}] \in \mathbb{R}^{n \times Mn}, C_i = [C_{i_1}, C_{i_2}, \ldots, C_{i_M}]^T \in \mathbb{R}^{Mn \times n},$$

for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$, $k = 0,\ 1,\ 2,\ \ldots, i \in \aleph$. For convenience, denote $\Lambda_i = K_i[\mathcal{D} \otimes I_n] C_i \in \mathbb{R}^{n \times n}$.

In this subsection, the output feedback control for DPSs will be reformulated as a delayed system in a unified framework, which makes it convenient to analyze the considered communication and feedback control scheme.

Throughout this paper, $t_k^i = t_k^j$ and $\tau_k^i = \tau_k^j$ are considered for $i \neq j$. Denote $\tau(t) = t - t_k$ for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$, $e_1(\vartheta, t_k) = \mathscr{M}(\vartheta, t_k) - \mathscr{M}(\bar{\vartheta}_i, t_k)$ for $\vartheta \in \Omega_i$. One obtains the following output-feedback controller form

$$u_i(\vartheta, t) = \Lambda_i \mathscr{M}(\vartheta, t - \tau(t)) - \Lambda_i e_1(\vartheta, t - \tau(t)), \tag{8}$$

for $\forall t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1}), k = 0,\ 1,\ 2,\ \ldots$, and $\vartheta \in \Omega_i, i \in \aleph$.

Remark 1. To design the networked sampled-data controller, one mentioned classical method is used widely in the last few decades, where the feedback controller is employed by $u(\vartheta, t_k) = KC\mathscr{M}(\vartheta, t_k)$ and the gain K should be designed such that the matrix $A + KC$ is Hurwitz, while it seems too conservative in that the sampling intervals should be sufficiently small for the obtained sampled-data control scheme, in other words, when the sampling intervals increases, the control scheme can be invalid. Moreover, it is difficult to get all the sampled-data $\mathscr{M}(\vartheta, t_k)$ due to its spatial properties. To compensate for the effect of controller discretization, in this paper, we choose the discrete time-delay approach which have been applied in [21–24] to deal with the sampled control problem.

Combining system (1) and controller (8), one has

$$\begin{aligned}\frac{\partial \mathscr{M}(\vartheta,t)}{\partial t} &= \Delta D(\vartheta)\mathscr{M}(\vartheta,t) + A\mathscr{M}(\vartheta,t) + E\nabla\mathscr{M}(\vartheta,t) + f(\mathscr{M}(\vartheta,t),t) \\ &\quad + \Lambda_i\mathscr{M}(\vartheta,t-\tau(t)) - \Lambda_i e_1(\vartheta,t-\tau(t)),\end{aligned} \tag{9}$$

for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ and $\vartheta \in \Omega_i, i \in \aleph$.

As shown in [25], there exist a continuable strong solution to the system (9) under the boundary conditions (2), (3) or (4) for $t \geq 0$ and any $\mathscr{M}(.,0) \in \mathcal{H}$ satisfying the corresponding boundary conditions.

3. Main results

Before the main results are given, we first introduce the following assumption and useful lemmas.

Assumption 1. There exists constant scalars f_m^i, f_M^i such that

$$f_m^i \leq \frac{f_i(\mathscr{M}(\vartheta,t),t) - f_i(\hat{\mathscr{M}}(\vartheta,t),t)}{\mathscr{M}(\vartheta,t) - \hat{\mathscr{M}}(\vartheta,t)} \leq f_M^i,$$

holds for any $\mathscr{M}(\vartheta,t), \hat{\mathscr{M}}(\vartheta,t) \in \mathbb{R}^n$, where $f(\cdot,0) = 0, \mathscr{M}(\vartheta,t) \neq \hat{\mathscr{M}}(\vartheta,t)$.

Lemma 1 [26]. *Let $a < b$, $S = S^T > 0$, then*

$$-\int_a^b \dot{\omega}^T(s)R\dot{\omega}(s) \leq -\frac{1}{b-a}\Upsilon^T diag\{S,\ 3S,\ 5S\}\Upsilon, \tag{10}$$

where

$$\Upsilon = [\Upsilon_1^T,\ \Upsilon_2^T,\ \Upsilon_3^T]^T,\ \Upsilon_1 = \omega(b) - \omega(a), \Upsilon_2 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)ds,$$

$$\Upsilon_3 = \omega(b) - \omega(a) - \frac{12}{(b-a)^2}\int_a^b (s - \frac{b+a}{2})\omega(s)ds.$$

Lemma 2 [27]. *If $f_1, f_2, \ldots, f_N : \mathbb{R}^m \to \mathbb{R}$ have positive value in an open set $\mathbf{D} \subset \mathbb{R}^m$, then the reciprocating convex combination of f_i on $\mathbf{D}$ satisfies*

$$\min\{\sum_{i=1}^N \frac{1}{\alpha_i} f_i(t)\} = \sum_{i=1}^N f_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t),$$

where $\alpha_i > 0$, $\sum_{i=1}^N \alpha_i = 1$,

$$\{g_{i,j} : \mathbb{R}^m \to \mathbb{R}, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{bmatrix} \geq 0\}.$$

Lemma 3 [28]. *If $0 < \delta_1 < 2\delta$ and let $V : [t_0 - \tau_M,\ \infty) \to [0,\ \infty)$ be an absolutely continuous function that satisfies*

$$\dot{V}(t) \leq -2\delta V(t) + \delta_1 \sup_{-\tau_M \leq \theta \leq 0} V(t+\theta), \tag{11}$$

for $t \geq t_0$, then

$$V(t) \leq exp^{-2\alpha(t-t_0)} \sup_{-\tau_M \leq \theta \leq 0} V(t_0+\theta), \quad \forall t \geq t_0, \tag{12}$$

where $\alpha > 0$ is a unique positive solution of

$$\alpha = \delta - \frac{\delta_1}{2} exp^{2\alpha\tau_M}. \tag{13}$$

Lemma 4 [29]. *Define a scalar function $x \in \mathcal{H}$. If $x(0) = 0$ or $x(l) = 0$, for a positive definite matrix $Q \in \mathbb{R}^n$,*

$$-\int_0^l \frac{dx(s)^T}{ds} Q \frac{dx(s)}{ds} ds \leq -\frac{\pi^2}{4l^2}\int_0^l x(s)^T Q x(s) ds.$$

Furthermore, if $x(0) = 0$ and $x(l) = 0$,

$$-\int_0^l \frac{dx(s)^T}{ds} Q \frac{dx(s)}{ds} ds \leq -\frac{\pi^2}{l^2}\int_0^l x(s)^T Q x(s) ds.$$

Lemma 5 [30]. *If $\phi : \mathbb{R} \to \mathbb{R}$ is a convex function, then for finite intervals $[a, b]$, integrable function $f : [a, b] \to \mathbb{R}$ satisfies*

$$\phi(\frac{1}{b-a}\int_a^b f(t)dt) \le \frac{1}{b-a}\int_a^b \phi(f(t))dt. \tag{14}$$

3.1. Design of sampling scheme with communication delay

Theorem 1. *Let $\gamma = \max_{i\in\aleph}(\vartheta_i - \vartheta_{i-1})$ and $D_0 = diag\{d_1^0, d_2^0, \ldots, d_n^0\}$. Given positive scalars $0 < 2\delta < \delta_1$, $\tau_M > 0$, if there exist symmetric positive definite matrices P_1, Q_0, Q_1, Q_2, and X, symmetric matrix P_2, and matrix G such that*

$$\mathcal{G} = \begin{bmatrix} \mathcal{Q} & G \\ * & \mathcal{Q} \end{bmatrix} > 0, \tag{15}$$

$$\Phi = [\phi_{ij}] - \frac{1}{\tau_M} exp^{-2\delta\tau_M}\Xi < 0, \tag{16}$$

hold, in which $\mathcal{Q} = diag\{Q_1, 3Q_1, 5Q_1\}$,

$$\begin{aligned}
\Xi &= \Xi_1^T\mathcal{Q}\Xi_1 + \Xi_1^T G\Xi_2 + \Xi_2^T G^T\Xi_1 + \Xi_2^T\mathcal{Q}\Xi_2,\\
\Xi_1 &= \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ I & I & -2I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ I & -I & 0 & -12I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\\
\Xi_2 &= \begin{bmatrix} 0 & I & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & I & 0 & 0 & I & -2I & 0 & 0 & 0 & 0 & 0\\ 0 & I & 0 & 0 & -I & 0 & -12I & 0 & 0 & 0 & 0 \end{bmatrix},\\
\phi_{11} &= A^T(P_1+P_2) + (P_1+P_2)A + 2\delta P_1 + Q_0\\
&\quad - \frac{\mu_f}{2}(F_M^T F_m + F_m^T F_M) - \frac{\chi_a\pi^2}{l^2}X, \ \phi_{12} = (P_1+P_2)\Lambda_i,\\
\phi_{18} &= -P_2 + A^T Q_2, \ \phi_{19} = P_1 + P_2 + \frac{\mu_f}{2}(F_M + F_m)^T,\\
\phi_{1,10} &= -(P_1+P_2)\Lambda_i, \quad \phi_{22} = -\delta_1 P_1, \phi_{28} = -\Lambda_i Q_2,\\
\phi_{55} &= -exp^{-2\delta\tau_M}Q_0, \phi_{88} = -2Q_2 + \tau_M Q_1, \quad \phi_{89} = Q_2,\\
\phi_{8,10} &= -Q_2\Lambda_i, \ \phi_{8,11} = Q_2 E, \ \phi_{99} = -\mu_f I,\\
\phi_{10,10} &= -\frac{\delta_1\pi^2}{4\gamma^2}(D_0Q_2 + Q_2D_0),\\
\phi_{11,11} &= -D_0(P_1+P_2) - (P_1+P_2)D_0 + \delta D(\vartheta)Q_2 + \delta Q_2 D(\vartheta) + X,
\end{aligned}$$

then, we have the following results:

a) *an unique strong solution to the Dirichlet boundary value problem (2), (8), (9) initialized with $\mathscr{M}(\cdot, 0) \in \mathcal{H}$ and $\chi_a = 1$ satisfies*

$$\begin{aligned}
&\int_{\Omega_i} \mathscr{M}^T(\vartheta,t)P_1\mathscr{M}(\vartheta,t)d\vartheta + \int_{\Omega_i}(\nabla\mathscr{M}(\vartheta,t))^T[D(\vartheta)Q_2 + Q_2D(\vartheta)](\nabla\mathscr{M}(\vartheta,t))d\vartheta\\
&\le exp^{-2\alpha(t-t_0)}\int_{\Omega_i}\Big(\mathscr{M}^T(\vartheta,t_0)(P_1+\tau_M Q_1)\mathscr{M}(\vartheta,t_0) + \frac{2\chi}{l}(\nabla\mathscr{M}(0,t_0))^T BD_0Q_2(\nabla\mathscr{M}(0,t_0))\\
&\quad + (\nabla\mathscr{M}(\vartheta,t_0))^T[D(\vartheta)Q_2 + Q_2D(\vartheta)](\nabla\mathscr{M}(\vartheta,t_0))\Big)d\vartheta,
\end{aligned} \tag{17}$$

with $\chi = 0$ and $\phi_{ij} = 0, i, j \in \{1, 2, \ldots, 11\}$ for $t \ge t_0$, where α is an unique positive solution of (13).

b) *if conditions of a) are satisfied with $\phi_{1,11} = (P_1+P_2)E$, $\phi_{1,12} = \frac{\pi^2}{4l^2}X$, $\phi_{12,12} = -\frac{\pi^2}{4l^2}X$, and else $\phi_{ij} = 0, i, j \in \{1, 2, \ldots, 12\}$, then an unique strong solution to the Neumann boundary value problem (3), (8), (9) initialized with $\mathscr{M}(\cdot, 0) \in \mathcal{H}$ and $\chi_a = \frac{1}{4}$ satisfies (17) with $\chi = 1$ for $t \ge t_0$ where α is an unique positive solution of (13).*

c) *if conditions of a) with $\phi_{ij} = 0, i, j \in \{1, 2, \ldots, 11\}$, and $-E(P_1+P_2) - (P_1+P_2)E + \delta D_0Q_2 + \delta Q_2D_0 \le 0$ hold, then an unique strong solution to the mixed boundary value problem (4), (8), (9) initialized with $\mathscr{M}(\cdot, 0) \in \mathcal{H}$ and $\chi_a = \frac{1}{4}$ satisfies (17) with $\chi = 1$ for $t \ge t_0$ where α is an unique positive solution of (13).*

Proof. We choose the following LKF:

$$V(t) = \sum_{i=1}^{5} V_i(t),$$

$$
\begin{aligned}
V_1(t) &= \int_\Omega \mathscr{M}^T(\vartheta,t)P_1\mathscr{M}(\vartheta,t)d\vartheta,\\
V_2(t) &= \int_\Omega\int_{t-\tau_M}^{t} exp^{2\delta(s-t)}\mathscr{M}^T(\vartheta,s)Q_0\mathscr{M}(\vartheta,s)dsd\vartheta,\\
V_3(t) &= \int_\Omega\int_{t-\tau_M}^{t}\int_\theta^t exp^{2\delta(s-t)}\left(\frac{\partial\mathscr{M}^T(\vartheta,s)}{\partial s}\right)Q_1\left(\frac{\partial\mathscr{M}(\vartheta,s)}{\partial s}\right)dsd\theta d\vartheta,\\
V_4(t) &= \int_\Omega(\nabla\mathscr{M}(\vartheta,t))^T[D(\vartheta)Q_2+Q_2D(\vartheta)](\nabla\mathscr{M}(\vartheta,t))d\vartheta\\
V_5(t) &= \chi(\mathscr{M}(0,t))^T[BD_0Q_2+Q_2D_0B](\mathscr{M}(0,t)),
\end{aligned}
\tag{18}
$$

with $P>0,\ Q_1>0,\ Q_2>0$; $\chi=0$ for boundary conditions (2) or (3) and $\chi=1$ for conditions (4). It should be mentioned that in sub-Lyapunov function $V_5(t)$, the term is chosen as $D(\vartheta)Q_2+Q_2D(\vartheta)$ with $D(\vartheta)Q_2=Q_2D(\vartheta)$.

The partial derivative of (18) along the solution of system (9) with respect to t can be achieved as follows:

$$
\begin{aligned}
\frac{\partial V_1(\vartheta,t)}{\partial t} &= \int_\Omega 2\frac{\partial\mathscr{M}^T(\vartheta,t)}{\partial t}P_1\mathscr{M}(\vartheta,t)d\vartheta\\
&= 2\sum_{i=1}^{N}\int_{\Omega_i}[\Delta D(\vartheta)\mathscr{M}(\vartheta,t)]^TP_1\mathscr{M}(\vartheta,t)+\nabla\mathscr{M}^T(\vartheta,t)E^TP_1\mathscr{M}(\vartheta,t)+\mathscr{M}^T(\vartheta,t)A^TP_1\mathscr{M}(\vartheta,t)\\
&\quad+f^T(\mathscr{M}(\vartheta,t),t)P_1\mathscr{M}(\vartheta,t)+\mathscr{M}^T(\vartheta,t-\tau(t))(\Lambda_i)^TP_1\mathscr{M}(\vartheta,t)-e_1(\vartheta,t-\tau(t))^T(\Lambda_i)^TP_1\mathscr{M}(\vartheta,t)d\vartheta.
\end{aligned}
\tag{19}
$$

In view of Assumption 1, denote

$$F_m=\mathrm{diag}\{f_m^1,\dots,f_m^n\},\ F_M=\mathrm{diag}\{f_M^1,\dots,f_M^n\}.$$

For any positive scalar $\mu_f>0$, one has

$$
\mu_f\begin{bmatrix}\mathscr{M}(\vartheta,t)\\ f(\mathscr{M}(\vartheta,t),t)\end{bmatrix}^T\Omega\begin{bmatrix}\mathscr{M}(\vartheta,t)\\ f(\mathscr{M}(\vartheta,t),t)\end{bmatrix}\le 0,
\tag{20}
$$

where $\Omega=\begin{bmatrix}\frac{1}{2}(F_m^TF_M+F_M^TF_m) & -\frac{1}{2}(F_m^T+F_M^T)\\ & I\end{bmatrix}$.

It is clear that

$$
\begin{aligned}
\frac{\partial V_2(t)}{\partial t}+2\delta V_2(t) &= \int_\Omega\mathscr{M}^T(\vartheta,t)Q_0\mathscr{M}^T(\vartheta,t)d\vartheta\\
&\quad-\int_\Omega exp^{-2\delta\tau_M}\mathscr{M}^T(\vartheta,t-\tau_M)Q_0\mathscr{M}^T(\vartheta,t-\tau_M)d\vartheta.
\end{aligned}
\tag{21}
$$

By using Lemma 1 and Lemma 2, one has

$$
\begin{aligned}
&-\int_\Omega\int_{t-\tau_M}^{t} exp^{2\delta(s-t)}\frac{\partial\mathscr{M}^T(\vartheta,s)}{\partial s}Q_1\frac{\partial\mathscr{M}(\vartheta,s)}{\partial s}d\vartheta\\
&\le -exp^{-2\delta\tau_M}\int_\Omega\int_{t-\tau_M}^{t}\frac{\partial\mathscr{M}^T(\vartheta,s)}{\partial s}Q_1\frac{\partial\mathscr{M}(\vartheta,s)}{\partial s}d\vartheta\\
&\le -\frac{1}{\tau_M}exp^{-2\delta\tau_M}\int_\Omega\Big[\frac{\tau_M}{\tau(t)}\xi^T(\vartheta,t-\tau(t),t)\mathcal{Q}\xi(\vartheta,t-\tau(t),t)\\
&+\frac{\tau_M}{\tau_M-\tau(t)}\xi^T(\vartheta,t-\tau_M,t-\tau(t))\mathcal{Q}\xi(\vartheta,t-\tau_M,t-\tau(t))\Big]d\vartheta\\
&\le -\frac{1}{\tau_M}exp^{-2\delta\tau_M}\int_\Omega\big[\xi^T(\vartheta,t-\tau(t),t)\mathcal{Q}\xi(\vartheta,t-\tau(t),t)\\
&+\xi^T(\vartheta,t-\tau_M,t-\tau(t))\mathcal{Q}\xi(\vartheta,t-\tau_M,t-\tau(t))\\
&+2\xi^T(\vartheta,t-\tau_M,t-\tau(t))G\xi(\vartheta,t-\tau(t),t)\big]d\vartheta,
\end{aligned}
\tag{22}
$$

with $\begin{bmatrix}\mathcal{Q} & G\\ * & \mathcal{Q}\end{bmatrix}\ge 0$, where

$$
\begin{aligned}
\xi(\vartheta,a,b) &= \begin{bmatrix}\xi_1^T(\vartheta,a,b) & \xi_2^T(\vartheta,a,b) & \xi_3^T(\vartheta,a,b)\end{bmatrix}^T,\\
\xi_1 &= \mathscr{M}(\vartheta,b)-\mathscr{M}(\vartheta,a),
\end{aligned}
$$

$$\xi_2 = \mathscr{M}(\vartheta, b) + \mathscr{M}(\vartheta, a) - \frac{2}{b-a}\int_a^b \mathscr{M}(\vartheta, s)ds,$$
$$\xi_3 = \xi_1 - \frac{12}{(b-a)^2}\int_a^b (s - \frac{a+b}{2})\mathscr{M}(\vartheta, s)ds.$$

Based on the above analysis (22), one has

$$\begin{aligned}\frac{\partial V_3(t)}{\partial t} + 2\delta V_3(t) \le& \int_\Omega \tau_M \frac{\partial \mathscr{M}^T(\vartheta,t)}{\partial t}Q_1\frac{\partial \mathscr{M}(\vartheta,t)}{\partial t}d\vartheta - \int_\Omega exp^{-2\delta\tau_M}\int_{t-\tau_M}^t \frac{\partial \mathscr{M}^T(\vartheta,s)}{\partial s}Q_1\frac{\partial \mathscr{M}(\vartheta,s)}{\partial s}d\vartheta \\ \le& \int_\Omega \tau_M\left(\frac{\partial \mathscr{M}^T(\vartheta,t)}{\partial t}\right)Q_1\left(\frac{\partial \mathscr{M}(\vartheta,t)}{\partial t}\right)d\vartheta \\ &-\frac{1}{\tau_M}exp^{-2\delta\tau_M}\int_\Omega \big[\xi^T(\vartheta, t-\tau(t), t)\mathcal{Q}\xi(\vartheta, t-\tau(t), t) \\ &+\xi^T(\vartheta, t-\tau_M, t-\tau(t))\mathcal{Q}\xi(\vartheta, t-\tau_M, t-\tau(t)) \\ &+2\xi^T(\vartheta, t-\tau_M, t-\tau(t))G\xi(\vartheta, t-\tau(t), t)\big]d\vartheta. \end{aligned} \quad (23)$$

Taking the partial derivative of $V_4(t)$ and $V_5(t)$,

$$\frac{\partial V_4(t)}{\partial t} + \frac{\partial V_5(t)}{\partial t} = -2\int_\Omega \frac{\mathscr{M}^T(\vartheta,t)}{\partial t}Q_2\Delta D(\vartheta)\mathscr{M}(\vartheta,t)d\vartheta. \quad (24)$$

In view of system (9), for a symmetric matrix P_2 and Q_2, one has

$$\begin{aligned}\sum_{i=1}^N\int_{\Omega_i}&\left(\mathscr{M}(\vartheta,t)P_2 + \frac{\partial\mathscr{M}(\vartheta,t)}{\partial t}Q_2\right)^T(\Delta D(\vartheta)\mathscr{M}(\vartheta,t) + E\nabla\mathscr{M}(\vartheta,t) + f(\mathscr{M}(\vartheta,t),t) + \Lambda_i\mathscr{M}(\vartheta,t-\tau(t)) \\ &+A\mathscr{M}(\vartheta,t) - \Lambda_i e_1(\vartheta,t-\tau(t)) - \frac{\partial\mathscr{M}(\vartheta,t)}{\partial t}\Bigg)d\vartheta = 0.\end{aligned} \quad (25)$$

By using integration by parts, under boundary conditions (2) or (3), the following relationships can be derived that

$$\begin{aligned}\int_\Omega \mathscr{M}^T(\vartheta,t)(P_1+P_2)\Delta D(\vartheta)\mathscr{M}(\vartheta,t)d\vartheta =& -\int_\Omega[\nabla\mathscr{M}(\vartheta,t)]^T(P_1+P_2)D(\vartheta)\nabla\mathscr{M}(\vartheta,t)d\vartheta \\ \le& -\int_\Omega[\nabla\mathscr{M}(\vartheta,t)]^T(P_1+P_2)D_0\nabla\mathscr{M}(\vartheta,t)d\vartheta.\end{aligned} \quad (26)$$

Moreover, under boundary conditions (4), the following relationship can be derived that

$$\begin{aligned}\int_\Omega \mathscr{M}^T(\vartheta,t)(P_1+P_2)\Delta D(\vartheta)\mathscr{M}(\vartheta,t)d\vartheta \le& -\mathscr{M}^T(0,t)(P_1+P_2)B(0)D\mathscr{M}(0,t) \\ &-\int_\Omega[\nabla\mathscr{M}(\vartheta,t)]^T(P_1+P_2)D_0\nabla\mathscr{M}(\vartheta,t)d\vartheta.\end{aligned} \quad (27)$$

According to the Wirtinger's inequality in Lemma 4, there exists a symmetric matrix Y such that

$$\begin{aligned}\frac{\pi^2}{(\vartheta_i-\vartheta_{i-1})^2}\int_{\vartheta_{i-1}}^{\vartheta_i} e_1^T(\vartheta,t-\tau(t))Ye_1(\vartheta,t-\tau(t))d\vartheta =& \frac{\pi^2}{(\vartheta_i-\vartheta_{i-1})^2}\int_{\vartheta_{i-1}}^{\frac{1}{2}(\vartheta_i+\vartheta_{i-1})} e_1^T(\vartheta,t-\tau(t))Ye_1(\vartheta,t-\tau(t))d\vartheta \\ &+\frac{\pi^2}{(\vartheta_i-\vartheta_{i-1})^2}\int_{\frac{1}{2}(\vartheta_i+\vartheta_{i-1})}^{\vartheta_i} e_1^T(\vartheta,t-\tau(t))Ye_1(\vartheta,t-\tau(t))d\vartheta \\ \le& \int_{\vartheta_{i-1}}^{\vartheta_i}\nabla\mathscr{M}^T(\vartheta,t-\tau(t))Y\nabla\mathscr{M}(\vartheta,t-\tau(t))d\vartheta.\end{aligned} \quad (28)$$

In addition, it is easy to obtain that

$$\begin{aligned}-\delta_1\sup_{\theta\in[-\tau_M,\ 0]}V(t+\theta) \le -\delta_1V(t-\tau(t)) \le& -\delta_1\sum_{i=1}^N\int_{\Omega_i}\mathscr{M}^T(\vartheta,t-\tau(t))P_1\mathscr{M}(\vartheta,t-\tau(t))d\vartheta \\ &-\delta_1\sum_{i=1}^N\int_{\Omega_i}\nabla\mathscr{M}^T(\vartheta,t-\tau(t))[D_0Q_2+Q_2D_0](\nabla\mathscr{M}(\vartheta,t-\tau(t)))d\vartheta.\end{aligned} \quad (29)$$

It is not difficult to obtain that

$$\sum_{i=1}^N\int_{\Omega_i}\mathscr{M}(\vartheta,t)(P_1+P_2)E\nabla\mathscr{M}^T(\vartheta,t)d\vartheta = \mathscr{M}(l,t)(P_1+P_2)E\mathscr{M}^T(l,t) - \mathscr{M}(0,t)(P_1+P_2)E\mathscr{M}^T(0,t). \quad (30)$$

From Lemma 4, for a positive definite matrix X,

$$-\frac{l^2}{\chi_a \pi^2}\int_\Omega \nabla \mathscr{M}^T(\vartheta,t) X \nabla \mathscr{M}(\vartheta,t) d\vartheta \le -\int_\Omega (\mathscr{M}(\vartheta,t)-\mathscr{M}(l,t))^T X(\mathscr{M}(\vartheta,t)-\mathscr{M}(l,t)) d\vartheta, \tag{31}$$

where $\chi_a = \frac{1}{4}$ is for boundary condition (3) or (4), and $\chi_a = 1$ is for boundary condition (2).

According to (20)–(31), it is derived that

$$\begin{aligned}\frac{\partial V(t)}{\partial t} + 2\delta V(t) - \delta_1 \sup_{\theta\in[-\tau_M,\ 0]} V(t+\theta) &\le \frac{\partial V(t)}{\partial t} + 2\delta V(t) - \delta_1 V(t-\tau(t)) \\ &\le \sum_{i=1}^{N}\int_{\Omega_i} \beta^T(\vartheta,t)\Phi\beta(\vartheta,t) d\vartheta,\end{aligned} \tag{32}$$

where $\beta(\vartheta,t)=\beta_a$ is for boundary condition (2) or (4), $\beta(\vartheta,t)=\beta_b$ is for boundary condition (3), $\beta_a(\vartheta,t)=\left[\beta_1^T\ \beta_2^T\ \beta_3^T\right]^T$, $\beta_b(\vartheta,t)=\left[\beta_1^T\ \beta_2^T\ \beta_4^T\right]^T$, and

$$\beta_1 = \left[\mathscr{M}^T(\vartheta,t) \quad \mathscr{M}^T(\vartheta,t-\tau(t)) \quad \tfrac{1}{\tau(t)}\int_{t-\tau(t)}^t \mathscr{M}^T(\vartheta,s)ds \quad \tfrac{1}{\tau(t)^2}\int_{t-\tau(t)}^t (s-\tfrac{2t-\tau(t)}{2})\mathscr{M}^T(\vartheta,s)ds\right]^T,$$

$$\beta_2 = \left[\mathscr{M}^T(\vartheta,t-\tau_M) \quad \tfrac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)} \mathscr{M}^T(\vartheta,s)ds \quad \tfrac{1}{(\tau_M-\tau(t))^2}\int_{t-\tau_M}^{t-\tau(t)} (s-\tfrac{2t-\tau(t)-\tau_M}{2})\mathscr{M}^T(\vartheta,s)ds\right]^T,$$

$$\beta_3 = \left[\tfrac{\partial \mathscr{M}^T(\vartheta,t)}{\partial t} \quad f^T(\mathscr{M}(\vartheta,t),t) \quad e_1^T(\vartheta,t-\tau(t))\right]^T,$$

$$\beta_4 = \left[\beta_3^T \quad \nabla\mathscr{M}^T(\vartheta,t) \quad \mathscr{M}^T(l,t)\right]^T.$$

In view of the condition (16) $\Phi < 0$, one has

$$\frac{\partial V(t)}{\partial t} \le -2\delta V(t) + \delta_1 \sup_{\theta\in[-\tau_M,\ 0]} V(t+\theta). \tag{33}$$

According to Lemma 1, Theorem 1 holds.

This proof is now completed. □

Remark 2. It should be noted that the Wirtinger's inequality was used in (53) given by [17] as follows

$$-2a_0(p_2-\delta p_3)\int_{\Omega_i}(\nabla\mathscr{M}(\vartheta,t))^2 d\vartheta \le -\frac{2a_0(p_2-\delta p_3)\pi^2}{bl^2}\int_{\Omega_i}(\mathscr{M}(\vartheta,t))^2 d\vartheta, \tag{34}$$

where $b=1$ when the boundary condition is Dirichlet condition; and $b=4$ when the boundary condition is mixed condition. Different from Fridman and Blighovsky [17], (28) in this paper applies the Wirtinger's inequality at each sampling interval. By using our method in this paper, the left side of (34) can be obtained

$$-2a_0(p_2-\delta p_3)\sum_{i=1}^{N}\int_{\Omega_i}(\nabla\mathscr{M}(\vartheta,t))^2 d\vartheta \le -\frac{2a_0(p_2-\delta p_3)\pi^2}{4\gamma^2}\sum_{i=1}^{N}\int_{\Omega_i}(\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta_{i-1},t))^2 d\vartheta, \tag{35}$$

where $\gamma \ge \vartheta_i - \vartheta_{i-1}$ is the maximum length of sampling interval in [17]. Compared with [17], our method is less conservative.

3.2. Design of sampling scheme with delay $\tau_k = 0$

Obviously, τ_M can be obtained by using Theorem 1 with communication delay. If the communication delay is not considered (i.e., $\tau_k = 0$), the updating interval is $t \in [t_k, t_{k+1})$ and the method given in Theorem 1 is then conservative. To design the sampling scheme without considering communication delay, the Lyapunov functional (18) can be modified as follows:

$$\begin{aligned}V_n(t) &= V(t) + V_d(t),\\ V_d(t) &= \int_\Omega\int_{t-\tau(t)}^t exp^{2\delta(s-t)}\mathscr{M}^T(\vartheta,s)Q_6\mathscr{M}(\vartheta,s)dsd\vartheta \\ &\quad - \frac{\pi^2}{4\tau_M^2}exp^{-2\delta\tau_M}\int_\Omega\int_{t-\tau(t)}^t (\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t)))^T Q_4(\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t)))dsd\vartheta.\end{aligned} \tag{36}$$

Remark 3. $V_d(t)$ is represented as a sum of the continuous time term $\int_\Omega\int_{t-\tau(t)}^t exp^{2\delta(s-t)}\frac{\partial\mathscr{M}^T(\vartheta,s)}{\partial s}Q_3\frac{\partial\mathscr{M}(\vartheta,s)}{\partial s}dsd\vartheta > 0$ with the discontinuous one (the second term of $V_d(t)$). By a simply computing, one has

$$V_d(t) \ge exp^{-2\delta\tau_M}\left(\int_\Omega\int_{t-\tau(t)}^t \frac{\partial\mathscr{M}^T(\vartheta,s)}{\partial s}Q_3\frac{\partial\mathscr{M}(\vartheta,s)}{\partial s}\right.$$

$$-\frac{\pi^2}{4\tau_M^2}[\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t))]^TQ_3[\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t))]\Bigg)dsd\vartheta.$$

Since $[\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t))]|_{s=t-\tau(t)}=0$, then by using the Wirtinger's inequality in Lemma 4, $V_d(t)\geq 0$.

Based on Lemma 5, one has

$$\begin{aligned}
&-\int_{t-\tau(t)}^{t}[\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t))]^TQ_3[\mathscr{M}(\vartheta,s)-\mathscr{M}(\vartheta,t-\tau(t))]ds\\
&\leq -\frac{1}{\tau(t)}[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))]^TQ_3[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))].
\end{aligned}\tag{37}$$

Taking the partial derivation of $V_d(t)$, one has

$$\begin{aligned}
\frac{\partial V_d(t)}{\partial t}+2\delta V_d(t)\leq&\int_\Omega\frac{\partial\mathscr{M}^T(\vartheta,t)}{\partial t}Q_3\frac{\partial\mathscr{M}(\vartheta,t)}{\partial t}d\vartheta\\
&-\frac{\pi^2}{4\tau_M^2}exp^{-2\delta\tau_M}\int_\Omega[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))]^TQ_3[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))]d\vartheta\\
&-\frac{\delta\pi^2}{2\tau_M^3}exp^{-2\delta\tau_M}[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))]^TQ_3[\mathscr{M}(\vartheta,t)-\mathscr{M}(\vartheta,t-\tau(t))].
\end{aligned}\tag{38}$$

Theorem 2. *Let $\gamma=\max_{i\in\aleph}(\vartheta_i-\vartheta_{i-1})$ and $D_0=diag\{d_1^0,\ d_2^0,\ \ldots,\ d_n^0\}$. Given positive scalars $0<\delta_1<\delta$, $\tau_M>0$, if there exist symmetric positive definite matrices P_1, Q_0, Q_1, Q_2, Q_3, X, symmetric matrix P_2, and matrix G such that*

$$\mathcal{G}_1=\begin{bmatrix}\mathcal{Q} & G\\ * & \mathcal{Q}\end{bmatrix}>0,\tag{39}$$

$$\hat{\Phi}=[\phi_{ij}]-\frac{1}{\tau_M}exp^{-2\delta\tau_M}\Xi<0,\tag{40}$$

where $\mathcal{Q}=diag\{Q_1,3Q_1,5Q_1\}$,

$$\begin{aligned}
\Xi&=\Xi_1^T\mathcal{Q}\Xi_1+\Xi_1^TG\Xi_2+\Xi_2^TG^T\Xi_1+\Xi_2^T\mathcal{Q}\Xi_2,\\
\Xi_1&=\begin{bmatrix}I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ I & I & -2I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ I & -I & 0 & -12I & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
\Xi_2&=\begin{bmatrix}0 & I & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & I & 0 & 0 & I & -2I & 0 & 0 & 0 & 0 & 0\\ 0 & I & 0 & 0 & -I & 0 & -12I & 0 & 0 & 0 & 0\end{bmatrix},\\
\phi_{11}&=A^T(P_1+P_2)+(P_1+P_2)A+2\delta P_1+Q_0-\frac{\chi_a\pi^2}{l^2}X\\
&\quad-\frac{\mu_f}{2}(F_M^TF_m+F_m^TF_M)-(\frac{(1+\frac{2\delta}{\tau_M})\pi^2}{2\tau_M^2})exp^{-2\delta\tau_M}Q_3,\\
\phi_{12}&=(P_1+P_2)\Lambda_i+(\frac{(1+\frac{2\delta}{\tau_M})\pi^2}{2\tau_M^2})exp^{-2\delta\tau_M}Q_3,\\
\phi_{18}&=-P_2+A^TQ_2,\ \ \phi_{19}=P_1+P_2+\frac{\mu_f}{2}(F_M+F_m)^T,\\
\phi_{1,10}&=-(P_1+P_2)\Lambda_i,\quad \phi_{28}=-\Lambda_iQ_2,\\
\phi_{22}&=-\delta_1P_1-(\frac{(1+\frac{2\delta}{\tau_M})\pi^2}{2\tau_M^2})exp^{-2\delta\tau_M}Q_3,\\
\phi_{55}&=-exp^{-2\delta\tau_M}Q_0,\ \ \phi_{88}=-2Q_2+\tau_MQ_1+Q_3,\\
\phi_{89}&=Q_2,\ \ \phi_{8,10}=-Q_2\Lambda_i,\ \ \phi_{8,11}=Q_2E,\ \ \phi_{99}=-\mu_fI,\\
\phi_{10,10}&=-\frac{\delta_1\pi^2}{4\gamma^2}(D_0Q_2+Q_2D_0),\\
\phi_{11,11}&=-D_0(P_1+P_2)-(P_1+P_2)D_0+\delta D(\vartheta)Q_2+\delta Q_2D(\vartheta)+X,
\end{aligned}$$

then, we have the following results:

a) *an unique strong solution to the Dirichlet boundary value problem (2), (8) and (9) initialized with* $\mathscr{M}(\cdot,0)\in\mathcal{H}$ *and* $\chi_a=1$ *satisfies*

$$\int_{\Omega_i}\mathscr{M}^T(\vartheta,t)P_1\mathscr{M}(\vartheta,t)d\vartheta+\int_{\Omega_i}(\nabla\mathscr{M}(\vartheta,t))^T[D(\vartheta)Q_2+Q_2D(\vartheta)](\nabla\mathscr{M}(\vartheta,t))d\vartheta$$
$$\leq exp^{-2\alpha(t-t_0)}V(t_0), \tag{41}$$

with $\chi=0$ *and* $\phi_{ij}=0, i,j\in\{1,2,\ldots,11\}$ *for* $t\geq t_0$ *where* α *is an unique positive solution of (13).*

b) *if conditions of a) are satisfied with* $\phi_{1,11}=(P_1+P_2)E$, $\phi_{1,12}=\frac{\pi^2}{4l^2}X$, $\phi_{12,12}=-\frac{\pi^2}{4l^2}X$, *and else* $\phi_{ij}=0, i,j\in\{1,2,\ldots,12\}$, *then an unique strong solution to the Neumann boundary value problem (3), (8), (9) initialized with* $\mathscr{M}(\cdot,0)\in\mathcal{H}$ *and* $\chi_a=\frac{1}{4}$ *satisfies (41) with* $\chi=1$ *for* $t\geq t_0$ *where* α *is an unique positive solution of (13).*

c) *if conditions of a) with* $\phi_{ij}=0, i,j\in\{1,2,\ldots,11\}$, *and* $-E(P_1+P_2)-(P_1+P_2)E+\delta D_0Q_2+\delta Q_2D_0\leq 0$ *hold, then an unique strong solution to the mixed boundary value problem (4), (8), (9) initialized with* $\mathscr{M}(\cdot,0)\in\mathcal{H}$ *and* $\chi_a=\frac{1}{4}$ *satisfies (41) with* $\chi=1$ *for* $t\geq t_0$ *where* α *is an unique positive solution of (13).*

Proof. Similar to proof of Theorem 1, one has

$$\frac{\partial V_n(t)}{\partial t}+2\delta V_n(t)-\delta_1\sup_{\theta\in[-\tau_M,\ 0]}V_n(t+\theta) \tag{42}$$

$$\leq\frac{\partial V_n(t)}{\partial t}+2\delta V_n(t)-\delta_1V_n(t-\tau(t)) \tag{43}$$

$$\leq\sum_{i=1}^N\int_{\Omega_i}\beta^T(\vartheta,t)\Phi\beta(\vartheta,t)d\vartheta, \tag{44}$$

where $\beta(\vartheta,t)$ is given in proof of Theorem 1.

This proof is now completed. □

Remark 4. In the proof of Theorem 2, since $\sup_{\theta\in[-\tau_M,\ 0]}V_n(t+\theta)>\sup_{\theta\in[-\tau_M,\ 0]}V(t+\theta)$, one has if

$$\dot{V_n}(t)+2\delta V_n(t)-\delta_1\sup_{-\tau_M\leq\theta\leq 0}V(t+\theta)\leq 0, \tag{45}$$

then the condition (11) holds, which means that

$$V_n(t)\leq exp^{-2\alpha(t-t_0)}\sup_{-\tau_M\leq\theta\leq 0}V_n(t_0+\theta),\quad\forall t\geq t_0.$$

4. Numerical example

In order to illustrate effectiveness of our results, the following example is given.

Consider the distributed parameter system as follows

$$\frac{\partial\mathscr{M}(\vartheta,t)}{\partial t}=\Delta D(\vartheta)\mathscr{M}(\vartheta,t)+E\nabla\mathscr{M}(\vartheta,t)+A\mathscr{M}(\vartheta,t)+f(\mathscr{M}(\vartheta,t),t)+\sum_{i=1}^N\chi_i(\vartheta)u_i(\vartheta,t), \tag{46}$$

where the domain of the space is $[0,\pi]$, which is divided into 10 sub-domains on average. For each sub-domain, there are 4 sensors to provide the measurement of the sampled point. The parameters of the systems (46) are given as: $u_i(\vartheta,t)=\Lambda_i\mathscr{M}(\bar{\vartheta}_i,t_k)$, $D(\vartheta)=0.5$, $E=0.1$, $A=-0.3$, $\Lambda_i=K_i(\mathcal{D}\otimes I)C_i=-0.1$, $I=1$, $C_i=\begin{bmatrix}1 & 2 & 1 & 0.5\end{bmatrix}^T$, $\mathcal{D}=\begin{bmatrix}1 & -0.1 & -0.8 & -0.1\\ -1 & 2 & -0.5 & -0.5\\ -0.5 & -0.5 & 1 & 0\\ 0 & -1 & -0.5 & 1.5\end{bmatrix}$.

Three boundary conditions for the distributed parameter system (46) are list as follows:

1. Dirichlet conditions
$$\mathscr{M}(0,t)=\mathscr{M}(\pi,t)=0,\quad t\in[0,+\infty); \tag{47}$$
2. Neumann conditions
$$\nabla\mathscr{M}(0,t)=\nabla\mathscr{M}(\pi,t)=0,\quad t\in[0,+\infty); \tag{48}$$
3. and mixed boundary conditions
$$\nabla\mathscr{M}(0,t)=\mathscr{M}(0,t),\quad\mathscr{M}(\pi,t)=0. \tag{49}$$

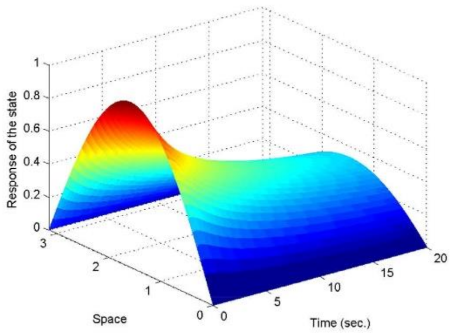


Fig. 3. Open loop profile of evolution of the system (46) with Dirichlet boundary conditions (47) and initial value $\mathscr{M}(\vartheta, 0) = sin(\vartheta)$.

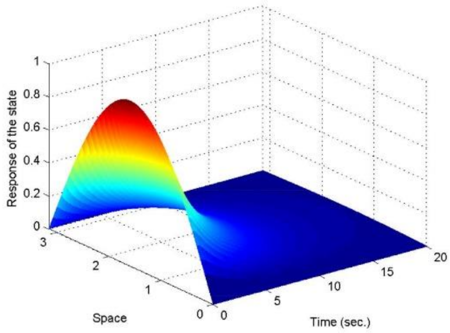


Fig. 4. Closed-loop profile of evolution of the system (46) with Dirichlet boundary conditions (47) and initial value $\mathscr{M}(\vartheta, 0) = sin(\vartheta)$.

Table 1
Maximum τ_M is derived with $\delta = 1$.

Method	Dirichlet (47)	Neumann (48)	Mixed condition (49)
[19]	0.3485	0.2451	0.1960
[17]	0.3955	–	0.2322
Theorem 1	0.4001	0.3443	0.2481
Theorem 2	0.7946	0.7210	0.5850

Table 2
Maximum δ is derived with $\tau_M = 0.196$ and $\delta_1 = 2\delta - 10^6$.

Method	Dirichlet (47)	Neumann (48)	Mixed condition (49)
[19]	1.7115	1.2890	1.0001
[17]	1.8018	–	1.2366
Theorem 1	1.8150	1.6676	1.2911
Theorem 2	6.2325	5.7927	3.6668

Comparison: Consider $f(\mathscr{M}(\vartheta, t), t) = 0.1tanh(\mathscr{M}(\vartheta, t))$. It is easy to obtain that $-F_m = F_M = 0.1$ due to $-\mathscr{M}(\vartheta, t) \leq \tanh(\mathscr{M}(\vartheta, t)) \leq \mathscr{M}(\vartheta, t)$. Choose $K_i = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix}$ and $\delta = 1$. The corresponding values of δ_1 are chosen to be close to 2δ (without loss of generality, we take $\delta_1 = 2\delta - 10^6$), which leads to a small decay rate α while large sampling intervals h_k.

Based on Theorem 1, the maximum of parameters τ_M are derived under the different value of the parameter δ. In view of Table 1, it is clear that τ_M is the biggest under Dirichlet condition (47), and τ_M is the least under mixed condition (49). Obviously, the results under mixed conditions (49) are comparatively conservative. As a comparison with [17,19] in Table 1, for the same δ, the maximum allowable delay τ_M obtained by using our method is the largest, which implies that our

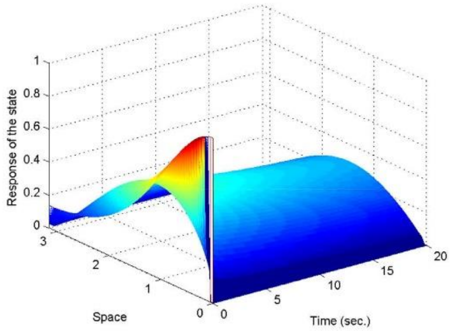


Fig. 5. Open-loop profile of evolution of the system (46) with Neumann boundary conditions (48) and initial value $\mathscr{M}(\vartheta, 0) = e^{\cos(\vartheta)-1}$.

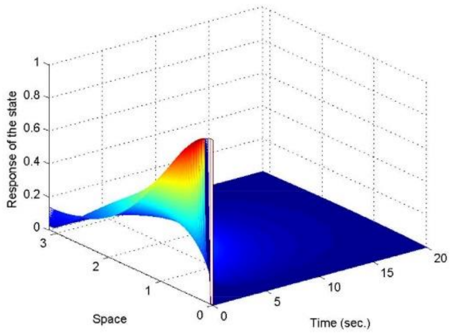


Fig. 6. Closed-loop profile of evolution of the system (46) with Neumann boundary conditions (48) and initial value $\mathscr{M}(\vartheta, 0) = e^{\cos(\vartheta)-1}$.

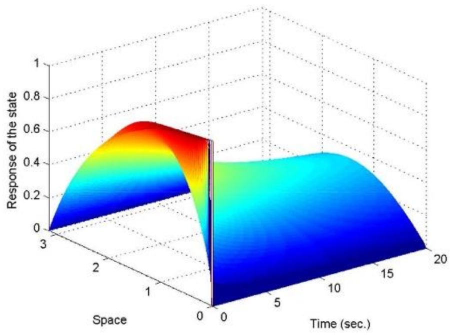


Fig. 7. Open-loop profile of evolution of the system (46) with mixed boundary conditions (49) and initial value $\mathscr{M}(\vartheta, 0) = \frac{e^{\pi}-e^{\vartheta}}{e^{\pi}-1}$.

results are less conservative and our method is superior. Maximum δ is derived in Table 2 by using Theorem 1, Theorem 2, [17,19], respectively. According to the Eq. (13), the bigger the parameter δ, the bigger decay rate α. In view of Table 2, the maximum parameter δ obtained by using our Theorems are the largest than ones in [17,19], which means that the results based on our method are to stabilize the system more quickly. It is also clear that our method is superior.

Simulation: Based on Theorem 1, choose $f(\mathscr{M}(\vartheta,t),t) = 0.8tanh(\mathscr{M}(\vartheta,t))$, $K_i = \begin{bmatrix}4 & 4 & 4 & 4\end{bmatrix}$ and $\delta = 1$. The numerical simulations of the DPSs (46) under different boundary conditions are discussed in the following. The profiles of evolution

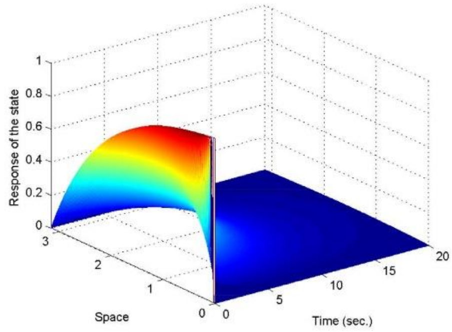


Fig. 8. Closed-loop profile of evolution of the system (46) with mixed boundary conditions (49) and initial value $\mathscr{M}(\vartheta, 0) = \frac{e^{\pi}-e^{\vartheta}}{e^{\pi}-1}$.

of open-loop system (46) are presented in Fig. 3, 5, 7 under different boundary and initial conditions, respectively. It is easy to see that the state of the system cannot converge quickly. The profiles of evolution of closed-loop system (46) are shown in Fig. 4, 6, 8 under different boundary and initial conditions, respectively. Simulations of solutions under the networked sampled-data controller (7) with $\vartheta_{i+1} - \vartheta_i = 0.1\pi$ $(i = 0,\ 1,\ 2,\ \ldots,\ 10)$ show that the state of the system can converge quickly, which means that effectiveness of the obtained exponentially stability conditions is illustrated.

5. Conclusion

In this paper, the distributed networked control problem is investigated for a class of DPSs governed by semilinear diffusion PDEs. The distributed sensor network is considered to provide the precise measurements, and a distributed networked sampled-data controller is designed to ensure the stabilization of the distributed parameter systems. Moreover, the time delays induced by the communication of the network is considered. To facilitate analysis of the closed-loop system with sampled-data, the time-delay dependent approach is used to reconstruct the closed-loop system. By using the Lyapunov method and some inequalities, the global exponential stability conditions are derived in terms of LMIs. Due to the introduce of the sensor network, in future studies, we will consider the event-based sampled-data control scheme to facilitate bandwidth reduction and to reduce network traffic burden. And the sensor networks with multiple communication channels studied in [2] is interesting, which will be one of the future research topics.

Declaration of Competing Interest

The authors declare that there are no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted. This work was supported in part supported by the NSERC.

CRediT authorship contribution statement

Huihui Ji: Project administration, Validation, Formal analysis, Validation. **Baotong Cui:** Project administration, Writing - review & editing. **Xinzhi Liu:** Formal analysis, Writing - review & editing.

References

[1] Peng C, Tian Y, Yue D. Output feedback control of discrete-time systems in networked environments. IEEE Trans Syst ManCybern-Part A 2010;41(1):185–90.
[2] Yin X, Li Z, Zhang L, Han M. Distributed state estimation of sensor-network systems subject to Markovian channel switching with application to a chemical process. IEEE Trans Syst Man Cybern 2018;48(6):864–74.
[3] Sakthivel R, Santra S, Kaviarasan B. Resilient sampled-data control design for singular networked systems with random missing data. J Franklin Inst 2018;355(3):1040–72.
[4] Lee SH, Park MJ, Kwon OM, Sakthivel R. A sampled-data control problem of neural-network-based systems using an improved free-matrix-based inequality. J Franklin Inst 2019;356(15):8344–65.
[5] Tarn T-J, Zavgren Jr JR, Zeng X. Stabilization of infinite-dimensional systems with periodic feedback gains and sampled output. Automatica 1988;24(1):95–9.
[6] Logemann H, Rebarber R, Townley S. Generalized sampled-data stabilization of well-posed linear infinite-dimensional systems. SIAM J Control Optim 2005;44(4):1345–69.
[7] Logemann H, Rebarber R, Townley S. Stability of infinite-dimensional sampled-data systems. Trans Am Math Soc 2003;355(8):3301–28.
[8] Rebarber R, Townley S. Stabilization of distributed parameter systems by piecewise polynomial control. IEEE Trans Autom Control 1997;42(9):1254–7.
[9] Logemann H. Stabilization of well-posed infinite-dimensional systems by dynamic sampled-data feedback. SIAM J Control Optim 2013;51(2):1203–31.

[10] Cheng M-B, Radisavljevic V, Chang C-C, Lin C-F, Su W-C. A sampled-data singularly perturbed boundary control for a heat conduction system with noncollocated observation. IEEE Trans Autom Control 2009;54(6):1305–10.
[11] Wang Z, Wu H. Finite dimensional guaranteed cost sampled-data fuzzy control for a class of nonlinear distributed parameter systems. Inf Sci 2016;327:21–39.
[12] Ghantasala S, El-Farra NH. Active fault-tolerant control of sampled-data nonlinear distributed parameter systems. Int J Robust Nonlinear Control 2012;22(1):24–42.
[13] Selivanov A, Fridman E. Sampled-data relay control of diffusion PDEs. Automatica 2017;82:59–68.
[14] Kang W, Fridman E. Distributed sampled-data control of Kuramoto-Sivashinsky equation. Automatica 2018;95:514–24.
[15] Wang J, Tsai S, Li H, Lam H. Spatially piecewise fuzzy control design for sampled-data exponential stabilization of semilinear parabolic PDE systems. IEEE Trans Fuzzy Syst 2018;26(5):2967–80.
[16] Wang J, Li H, Wu H. A membership-function-dependent approach to design fuzzy pointwise state feedback controller for nonlinear parabolic distributed parameter systems with spatially discrete actuators. IEEE Trans Syst Man Cybern 2016;47(7):1486–99.
[17] Fridman E, Blighovsky A. Robust sampled-data control of a class of semilinear parabolic systems. Automatica 2012;48(5):826–36.
[18] Liu X, Zhang K, Xie W. Pinning impulsive synchronization of reaction-diffusion neural networks with time-varying delays. IEEE Trans Neural Netw LearnSyst 2016;28(5):1055–67.
[19] Selivanov A, Fridman E. Distributed event-triggered control of diffusion semilinear PDEs. Automatica 2016;68:344–51.
[20] Curtain RF, Zwart H. An introduction to infinite-dimensional linear systems theory, vol 21. Springer Science & Business Media; 2012.
[21] Fridman E, Seuret A, Richard J-P. Robust sampled-data stabilization of linear systems: an input delay approach. Automatica 2004;40(8):1441–6.
[22] Gao H, Chen T, Lam J. A new delay system approach to network-based control. Automatica 2008;44(1):39–52.
[23] Wei Y, Qiu J, Karimi HR, Ji W. A novel memory filtering design for semi-Markovian jump time-delay systems. IEEE Trans. Syst. Man Cybern 2017;48(12):2229–41.
[24] Wei Y, Qiu J, Shi P, Wu L. A piecewise-Markovian Lyapunov approach to reliable output feedback control for fuzzy-affine systems with time-delays and actuator faults. IEEE Trans Cybern 2017;48(9):2723–35.
[25] Fridman E. Sampled-data distributed H_∞ control of transport reaction systems. SIAM J Control Optim 2013;51(2):1500–27.
[26] Kim J-H. Further improvement of Jensen inequality and application to stability of time-delayed systems. Automatica 2016;64:121–5.
[27] Park P, Ko JW, Jeong C. Reciprocally convex approach to stability of systems with time-varying delays. Automatica 2011;47(1):235–8.
[28] Halanay A, Halanay A. Differential equations: stability, oscillations, time lags, vol 6. Elsevier; 1966.
[29] Hardy GH, Littlewood JE, Pólya G, et al. Cambridge mathematical library Inequalities. Cambridge University Press; 1988.
[30] Chen J, Xu S, Chen W, Zhang B, Ma Q, Zou Y. Two general integral inequalities and their applications to stability analysis for systems with time-varying delay. Int J Robust Nonlinear Control 2016;26(18):4088–103.

南京理工大学图书馆

检 索 报 告

委托人： 季慧慧

委托人单位： 南京审计大学

检索要求： 季慧慧以第一作者发表的论文:1、在 SCIE 数据库中被收录情况；2、论文发表期刊的分区情况

检索数据库： SCIE（科学引文索引扩展版）
中国科学院文献情报中心期刊分区表（2021）

检索时间： 2023 年 8 月 27 日

检 索 结 果

本次检索对季慧慧提供的论文在 SCIE 数据库中被收录情况及论文发表期刊在中国科学院文献情报中心期刊分区表上的分区情况进行了检索，结果如下：

1、收录情况

季慧慧以第一作者发表的论文被 SCIE 数据库收录 2 篇。

2、分区情况

其论文发表期刊：1、*COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION* 在中科院文献情报中心期刊分区表(2021)的数学(大类）学科中为 1 区 TOP 期刊；2、*IET CONTROL THEORY AND APPLICATIONS* 在中科院文献情报中心期刊分区表（2021）的工程技术（大类）学科中为 3 区期刊。

检索结果详见附件（共计 3 页）。

特此证明！

检索证明人：龚凯丽　　审核人：周莹莹

教育部科技查新工作站（L28）
南京理工大学图书馆
2023 年 8 月 27 日

210094 南京市孝陵卫 200 号　Tel：025-84314962　Email:tsgxx@mail.njust.edu.cn

Web of Science™

2 record(s) printed from Clarivate Web of Science

第 1 条，共 2 条

标题: Networked sampled-data control of distributed parameter systems via distributed sensor networks

作者: Ji, HH (Ji, Huihui); Cui, BT (Cui, Baotong); Liu, XZ (Liu, Xinzhi)

来源出版物: COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION **卷:** 98 **文献号:** 105773 **DOI:** 10.1016/j.cnsns.2021.105773 **提前访问日期:** FEB 2021 **出版年:** JUL 2021

Web of Science 核心合集中的 "被引频次": 6

被引频次合计: 6

入藏号: WOS:000635186100001

文献类型: Article

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Math & Stat, 86 West Yushan Rd, Nanjing 211815, Peoples R China.
[Cui, Baotong] Jiangnan Univ, Minist Educ, Key Lab Adv Proc Control Light Ind, Wuxi 214122, Jiangsu, Peoples R China.
[Cui, Baotong] Jiangnan Univ, Sch IoT Engn, Wuxi 214122, Jiangsu, Peoples R China.
[Liu, Xinzhi] Univ Waterloo, Dept Appl Math, Waterloo, ON N2L 3G1, Canada.

通讯作者地址: Ji, HH (通讯作者)，Nanjing Audit Univ, Sch Math & Stat, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

电子邮件地址: jihuihui2020@nau.edu.cn; btcui@jiangnan.edu.cn; xzliu@uwaterloo.ca

---

第 2 条，共 2 条

标题: Event-triggered predictor-based control of distributed parameter systems

作者: Ji, HH (Ji, Huihui); Cui, BT (Cui, Baotong); Liu, XZ (Liu, XinZhi)

来源出版物: IET CONTROL THEORY AND APPLICATIONS **卷:** 15 **期:** 5 **页:** 721-736 **DOI:** 10.1049/cth2.12073 **提前访问日期:** DEC 2020 **出版年:** MAR 2021

Web of Science 核心合集中的 "被引频次": 1

被引频次合计: 1

入藏号: WOS:000604125800001

文献类型: Article

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.
[Liu, XinZhi] Univ Waterloo, Sch Appl Math, Waterloo, ON N2L 3G1, Canada.

通讯作者地址: Ji, HH (通讯作者)，Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

电子邮件地址: jihuihui2009@163.com

End of File

期刊全称：	Communications in Nonlinear Science and Numerical Simulation					
期刊简称：	COMMUN NONLINEAR SCI		ISSN：	1007-5704		
年份：	2021年		综述：	否		

	学科名称	分区	Top期刊
小类	MATHEMATICS, APPLIED应用数学	1	-
小类	MATHEMATICS, INTERDISCIPLINARY APPLICATIONS数学跨学科应用	1	-
小类	MECHANICS力学	2	-
小类	PHYSICS, FLUIDS & PLASMAS物理：流体与等离子体	1	-
小类	PHYSICS, MATHEMATICAL物理：数学物理	1	-
大类	数学	1	是

期刊影响因子				总被引频次		
2018年	2019年	2020年	2018-2020年平均	2019年	2020年	2019年-2020年
3.967	4.115	4.260	4.114	12329	14765	27094

备注：

说说您的看法（AddJComment.aspx?title=Communications+in+Nonlinear+Science+and+Numerical+Simulation&issn=1007-5704&year=2021&department=南京理工大学&ip=112.25.223.102)

科研项目 高教项目 科研成果 学术活动 高被引 校级项目申报 纵向项目申报 教研项目申报 入账办理 更多 季慧慧

成果列表

考核时间更新 新增 待确认网推论文

名称 模糊 审核状态 所属单位 第一作者姓名 模糊 更多 查询

名称	所有作者	成果日期	成果类型	审核状态	操作
Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems	季慧慧,徐胜元（外）,张贺（外）	2024-01-01	论文	学校通过	收录补录 转载补录
Networked sampled-data control of distributed parameter systems via distributed sensor networks SCIE	季慧慧,Cui, Baotong（外）,Liu, Xinzhi（外）	2021-07-01	论文	学校通过	收录补录 转载补录
Event-triggered predictor-based control of distributed parameter systems	季慧慧,崔宝同（外）,刘新芝（外）	2021-03-01	论文	学校通过	收录补录 转载补录
2023年度江苏省自动化学会科学技术奖	张贺（外）,喻果（外）,黄书华（外）,季慧慧,彭岳（外）,薄翠梅（外）	2024-02-23	获奖	学校通过	
Finite-time quasi- synchronization of stochastic multilayer networks with energy consumption estimation	徐玉华,Wu Xiaoqun（外）,Wan Xiaoxiao（外）,季慧慧,周世超	2022-04-16	论文	学校通过	
Event-generator-based H∞ control of fuzzy distributed parameter systems	季慧慧,崔宝同（外）	2022-03-28	论文	学校通过	收录补录 转载补录
概率论与数理统计课程思政的探索与思考 知	季慧慧	2022-01-05	论文	学校通过	收录补录 转载补录

成果查看

基本信息 收录转载 依托项目 审核记录 修改记录

基本信息

名称	Networked sampled-data control of distributed parameter systems via distributed sensor networks		
发表/出版时间	2021-07-01	发表/刊物论文集	Communications in Nonlinear Science and Numerical Simulation
教育部统计归属	科技类	期刊来源类型	外文期刊
论文类型	期刊论文	所属单位	统计与数据科学学院
刊物级别	SCI 一区		
SCI论文分区		SSCI分区	
影响因子		项目来源	其他课题
一级学科	数学	奖励级别	特别奖励期刊
字数	1 万字	刊型	正刊
是否符合审计提升	否	提升后级别	特别奖励期刊
文献类型			

详细信息

发表范围	国外学术期刊	学校署名	第一单位
卷期页	卷: 98	DOI	10.1016/j.cnsns.2021.105773
ISSN号	1007-5704	CN号	
附件	Commun Non...imulat2021.pdf 下载 预览		

关闭

3. Event-generator-based H∞ control of fuzzy distributed parameter systems (中科院 1 区 TOP 期刊、特别奖励期刊) (1/2)

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 432 (2022) 28–49

FUZZY sets and systems

www.elsevier.com/locate/fss

Event-generator-based $H\infty$ control of fuzzy distributed parameter systems

Huihui Ji [a,*], Baotong Cui [b,c]

[a] School of Statistics and Mathematics, Nanjing Audit University, Nanjing 211815, China

[b] Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, China

[c] School of IoT Engineering, Jiangnan University, Wuxi 214122, China

Received 30 September 2020; received in revised form 21 March 2021; accepted 22 March 2021

Available online 1 April 2021

Abstract

This paper studies the adaptive event-based H_∞ control problem for a class of T-S fuzzy distributed parameter system with parameter uncertain and actuator faults. To overcome the drawback of the period control scheme, an event-triggered control scheme with adaptive threshold is proposed to minimize the number of unnecessary sampled data transmission and to reduce the update frequency of the controller. Furthermore, a T-S fuzzy controller based on the adaptive event-triggered sampled data is designed to ensure the stochastic exponential stability of the distributed parameter system with H_∞ disturbance attenuation performance. Based on a new Lyapunov functional and inequality technique, the stochastic exponential stability criterion of the closed-loop system is obtained, and the controller parameters are designed. The new developed inequality can reduce the conservatism of the stability criterion. Finally, the effectiveness of the theoretical calculation results is verified by numerical simulation, and the results are compared with the relevant literature in the simulation, showing that the methods and results in this paper are less conservative.

Keywords: Distributed parameter systems; Adaptive event-triggered control; H_∞ control; T-S fuzzy model; Actuator fault; Stochastic exponential stability

1. Introduction

In engineering systems, nonlinearity is inevitable, that is, the relationship between the state, input and output of the system is not all linear, and the mutual coupling between the elements makes the stability analysis, control performance realization and optimization of the system more complex and difficult. Based on the fuzzy set, in 1985, Takagi and Sugeno [1] proposed a Takagi-Sugeno (T-S) fuzzy model to deal with nonlinear processes. The T-S fuzzy model connects the theory of fuzzy logic with the strict mathematical theory of nonlinear systems or linear systems, and provides an effective thinking framework for the local linearization analysis of nonlinear systems, thus realizing

* Corresponding author at: School of Statistics and Mathematics, Nanjing Audit University, 86 West Yushan Road, Nanjing 211815, China.
E-mail addresses: jihuihui2009@163.com (H. Ji), btcui@jiangnan.edu.cn (B. Cui).

https://doi.org/10.1016/j.fss.2021.03.012

the extension and application of the control theory of linear systems to the control of nonlinear systems [2]. In recent decades, the rapid development of fuzzy control theory has been witnessed with abundant research results [3], [4], [5]. The fuzzy control theory of lumped parameter systems is quite mature, however, the fuzzy control theory of distributed parameter systems is still in the initial stage due to its complex spatial characteristics. How to extend the techniques and methods of lumped parameter systems to the analysis and control of distributed parameter systems has attracted the attention of increasing researchers. Subsequently, T-S fuzzy model was assumed to accurately describe a class of nonlinear distributed parameter systems in [6], [7], where a fuzzy boundary controller and a pointwise fuzzy controller based on continuous time were designed respectively in [6] and [7]. Considering the actual situation may be that the measuring tool is not accurate enough, the working mechanism of the controlled object is not clear, or the system is affected by the exogenous input representing the disturbances, measurement noises, etc. The uncertainty or exogenous input of the system will become cumulatively serious with the increasing complexity of the object. Thus, it is of theoretical significance and practical value to study the disturbance suppression control of fuzzy model of distributed parameter system with uncertain terms. As is well known, H_∞ control method provides an effective way to solve the robust control of, which can guarantee the stability of the system, meanwhile restrain the influence of disturbance on the system performance below a certain level, so as to realize the robustness of the controlled object about external disturbance. For distributed parameter systems, H_∞ control problems have been studied since the late 1980s [8], [9], [10], [11]. This paper mainly focus on the H_∞ control problems for a class of T-S fuzzy distributed parameter system with uncertainty and exogenous disturbance.

To improve the control performance of the system, the advanced digital technology has been widely used in various practical systems with its characteristics of flexibility, efficiency and easy maintenance. This has stimulated the study of sampled data controller, which can control the continuous object system through a discrete time controller (a controller based on sampled data). Compared with the continuous time controller, the controller based on the sampled data can reduce the implementation cost and time through the microcontroller or digital computer, and this kind of controller has the advantages of high precision, reliability, effective interference suppression and strong universality. Therefore, the study of sampled data control methods has become one of the hot topics. A sampled-data static output feedback controller was suggested in [12] to deal with the sampled-data distributed H_∞ control problem for transport reaction systems. Based on the results in [12], a periodic Round-Robin scheduling protocol was used in [13] to handle network-based H_∞ filtering for a class of parabolic distributed parameter systems. In the studies [12], [13], Lyapunov method is employed which contributes to derive the stability of the system and the corresponding disturbance attenuation performance. To reduce the conservatism of the above results, an improved H_∞ sampled-data control method was proposed in [14] for a class of semilinear parabolic distributed parameter systems. Considering the fact that the sampled-data control schemes in [12], [13], [14] are difficult to implement and with a high cost because the acquisition of accurate average sampling data requires a large number of spatial point sampling measurements. The finite number of point spatial state measurements and Razumikhin-type approach were used in [15] to solve the sampled-data distributed H_∞ control problem for a class of semilinear distributed parameter systems. It should be pointed out that the sampled-data control strategies mentioned in above works are based on the periodic control method, which can result in unnecessary information transfer and resource waste in that the sampled-data is still transmitted and the controller is still updated even when the system reaches an desired state. Therefore, how to overcome the drawback of the periodic sampled-data control method to solve the robust control problem is a topic worth studying.

Event-triggered control scheme provides an aperiodic way to perform control tasks, that is, it is only related to the sampled-data information at the sampling time, and it can judge whether the sampled-data signal at the sampling time is sent to the controller according to the predefined event-triggered conditions. Only when some performance indicators of the system exceed the expected value, the sampled data will be sent and the control signal will be updated to achieve a satisfactory control performance. Event-triggered control scheme can operate without the additional monitoring hardware, so it is economical and convenient to use. Compared with the traditional time-triggered control methods, the event-triggered control scheme can effectively reduce the consumption of control energy and the consumption of transmission bandwidth, which has aroused the interest and attention of many experts and engineers in recent years [16], [17]. Particularly, since the serving Åström and Arzén [18], [19] have demonstrated the advantages of event-triggered control in reducing the consumption of bandwidth and data transfer for sampling through specific experimental comparison. Event-triggered control, as an event-driven aperiodic control signal update strategy, has been widely studied in lumped parameter systems, however, the event-based sampled-data control problems of distributed parameter systems are poorly studied and urgently needs to be solved. Recently, a finite-time event-triggered reliable fuzzy controller was adopted in [20] for a class of nonlinear distributed parameter systems with actuator faults and with H_∞ disturbance attention performance. Considering that the preset constant threshold value of event-triggering control schemes may not be suitable for the complex system structure, whereas the above work [20] is based on the time-invariant threshold value. Therefore, it is necessary to design an adaptive threshold event-triggered controller to adapt to the change of the system.

Motivated by the aforementioned discussions, this study focuses on adaptive event-triggered H_∞ control problem of a class of T-S fuzzy distributed parameter systems with parameter uncertainties and actuator faults. To overcome the drawback of the period control scheme and the event-triggered control scheme with constant threshold, an adaptive event-triggered control scheme is proposed to minimize the number of unnecessary sampled data transmission and to reduce the update frequency of the controller. Furthermore, a T-S fuzzy controller based on the adaptive event-triggered sampled data is designed to ensure the stochastic exponential stability of the distributed parameter system with H_∞ disturbance rejection performance. Based on a new Lyapunov functional and inequality technique, the stochastic exponential stability criterion of the closed-loop system is obtained, and the controller parameters are designed. The new developed inequality can reduce the conservatism of the stability criterion. Finally, the effectiveness of the theoretical calculation results is verified by numerical simulation, and the results are compared with the relevant literature in the simulation, showing that the methods and results in this paper are less conservative.

Notation. $\mathbb{R}$ presents the set of all real numbers. $\mathbb{R}^n$ denotes the set of all n dimensional column vectors. $\mathbb{R}^{n\times m}$ states the set of all $n\times m$ matrices. For $s\in[l_1,l_2]$, $\mathcal{H}^1(l_1,l_2)$ stands for the Sobolev space of absolutely continuous functions $x:\ [l_1,l_2]\times\mathbb{R}\to\mathbb{R}^n$ with satisfying $\frac{dx}{ds}$ being the Hilbert space of square integrable functions. $\|\ w(x,t)\ \|_{\mathcal{L}^2}=\sqrt{\int_a^b w^T(x,t)w(x,t)dx}<\infty$. diag$\{\cdots\}$ presents a block-diagonal matrix. $[a_{ij}]_{m\times n}$ means the matrix of elements a_{ij}, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$. $\mathbf{E}w$ stands for the mathematical expectation of the variable w. $\min\{A\}$ presents the smallest element taken from the set A. $\max\{A\}$ presents the largest element taken from the set A. $\mathbb{N}$ is the set of all integers. $\begin{bmatrix} P_1 & P_2^T \\ * & P_3 \end{bmatrix}=\begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}$

2. Preliminaries

Consider a class of T-S fuzzy distributed parameter systems with parameter uncertainty

Rule i: IF $\theta_1(x,t)$ is χ_1^i, $\theta_2(x,t)$ is χ_2^i, $\cdots$, $\theta_\zeta(x,t)$ is χ_ζ^i, THEN

$$\frac{\partial w(x,t)}{\partial t}=\frac{\partial}{\partial x}(D(x)\nabla w(x,t))+(A_i(x)+A_{ui}(x,t))w(x,t)+B_i(x)u(x,t)$$
$$+C_i(x)\upsilon(x,t),\quad i\in\{1,2,\cdots,s\},\tag{1}$$

where $\theta_1(x,t)$, $\cdots$, $\theta_\zeta(x,t)$ stands for the premise variables; χ_1^i, χ_2^i, $\cdots$, $\chi_\zeta^i(x,t)$ presents the fuzzy sets, i states the i_{th} fuzzy rules, $i\in\{1,2,\cdots,r\}$ and r is the number of the fuzzy rules. $w(x,t)\in\mathbf{R}^n$ is the state variable, $x\in\Omega=[\underline{x},\ \overline{x}]$ stands for position variable, $t\geq 0$ is the time variable, the diffusion coefficient $D(x)=\mathrm{diag}\{d_1(x),d_2(x),\cdots,d_n(x)\}$ and $d_i(x)$, $i=1,2,\cdots,n$ are continuous real functions with first derivatives, $0<d^0<d_i(x)$, $i=1,2,\cdots,n$. $\upsilon(x,t)\in\mathbf{R}^{n_\upsilon}$ is the external disturbance satisfying $\int_0^\infty \upsilon(x,t)^T\upsilon(x,t)dt<\infty$. $u(x,t)\in\mathbf{R}^{n_u}$ is the control input. $A_i(x)$, $B_i(x)$ and $C_i(x)$ are given matrices associated with x and with appropriate dimensions.

The i_{th} local model of system (1) is defined as $(A_i(s),\ B_i(s))$, and $A_{ui}(x,t)$ is the uncertainty parameter and satisfies

$$A_{ui}(x,t)=M_i(x)F_i(x,t)N_i(x),\quad i\in\{1,2,\cdots,r\},\tag{2}$$

where $M_i(x)$, $N_i(x)$ is the known real variable for the position x, $F_i(x,t)$ is the unknown time-varying function and satisfies

$$F_i(x,t)^TF_i(x,t)\leq I,\quad i\in\{1,2,\cdots,r\}.\tag{3}$$

Let $\theta(x,t)=\begin{bmatrix}\theta_1(x,t) & \cdots & \theta_r(x,t)\end{bmatrix}$, and define

$$\mu_i(\theta(x,t))=\frac{\prod_{j=1}^r\mu_{ij}(\theta_j(x,t))}{\sum_{k=1}^r\prod_{j=1}^r\mu_{kj}(\theta_j(x,t))},\ \sum_{i=1}^r\mu_i(\theta(x,t))=1,$$

where $\mu_{ij}(\theta_j(x,t))$ is the membership of the variable $\theta_j(x,t)$ in the fuzzy set χ_j^i. For convenience, denote $\mu_i = \mu_i(\theta(x,t))$.

The global fuzzy model based on the weight mean value is given as follows:

$$\frac{\partial w(x,t)}{\partial t} = \frac{\partial}{\partial x}(D(x)\nabla w(x,t)) + \sum_{i=1}^{r}\mu_i(\theta(x,t))(A_i(x) + A_{ui}(x,t))w(x,t) + \sum_{i=1}^{r}\mu_i(\theta(x,t))B_i(x)u(x,t) + \sum_{i=1}^{r}\mu_i(\theta(x,t))C_i(x)\upsilon(x,t). \tag{4}$$

For distributed parameter system (4), the boundary condition is given as

$$\nabla w(\underline{x},t) = Ew(\underline{x},t), \tag{5}$$

$$w(\overline{x},t) = 0,\ t \geq 0, \tag{6}$$

and initial condition is given as

$$w(x,0) = w_0(x),\ x \in [\underline{x},\overline{x}], \tag{7}$$

where $w_0(x) \in \mathcal{H}^1(\underline{x},\overline{x})$, E is a given real number.

2.1. A fuzzy controller based on adaptive event-triggered sampled data

Firstly, the spatial region of the system $[\underline{x},\overline{x}]$ is divided evenly into subintervals of ℓ, and each subinterval is represented as $[x_{l-1},x_l)$, $\hat{x}_l$ is the midpoint of the subinterval $[x_{l-1},x_l)$; t_kh is aperiodic sampling instants, where $0 \leq t_0 < t_1 < \cdots < t_k$ is a series of non-negative integers and $\lim_{k\to\infty} t_kh = \infty$.

In this paper, there exists a communication network between the sampler and the controller, and the time delay τ_k will be induced when the first t_k data passes through the network. Then, the update interval control input is $[t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$.

Without changing the communication network, a component-based event triggering scheme can be introduced to improve the communication performance of the network. To this end, the sampled-data based on the self-triggered scheme [30], discrete event-triggered scheme (DETS) [31] and dynamic DETS [32,33] were proposed, respectively. It is worth mentioning that there is no Zeno behavior [34] with the DETS [31]. However, authors in [35] has pointed out that there are still a lot of packets over the network when the system is stable. To further improve the communication performance, the DETS with an adaptive threshold is proposed in [22]. Adjusting the triggering threshold online, the DETS is more effective than the DETS with a constant threshold.

Based on the above analysis, the DETS with an adaptive threshold is introduced as follows:

$$\sum_{l=1}^{\ell} e_t(\hat{x}_l,t)^T\Omega_l e_t(\hat{x}_l,t) \leq \rho(\ell_{ki})\sum_{l=1}^{\ell} w(\hat{x}_l,t_kh)^T\Omega_l w(\hat{x}_l,t_kh), \tag{8}$$

$$\rho(\ell_{ki}) = \min\{\rho_0,\ \rho_0 e^{1-\frac{4}{\pi}actan\left(\frac{1+\sum_{l=1}^{\ell} w(\hat{x}_l,\ell_{ki})^T w(\hat{x}_l,\ell_{ki})}{1+\sum_{l=1}^{\ell} w(\hat{x}_l,t_kh)^T w(\hat{x}_l,t_kh)}\right)}\}, \tag{9}$$

where h is the sampling period of the periodic sampler, $e_t(\hat{x}_l,t) = w(\hat{x}_l,\ell_{ki}) - w(\hat{x}_l,t_kh)$, $\ell_{ki} = t_kh + ih$, $i = 1,2,\cdots,n_k$, and n_k stands for the sampling number during the update period $[t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$ of control input, $\rho_0 \in [0,1)$ and symmetric positive definite matrices Ω_l, $(l = 1,2,\cdots,\ell)$ will be given later.

t_kh is the last event-triggered instant, then the next event-triggered instant is

$$t_{k+1}h = t_kh + \max_i\{ih|\sum_{l=1}^{\ell} e_t(\hat{x}_l,t)^T\Omega_l e_t(\hat{x}_l,t) \leq \rho(\ell_{ki})\sum_{l=1}^{\ell} w(\hat{x}_l,t_kh)^T\Omega_l w(\hat{x}_l,t_kh)\}. \tag{10}$$

Remark 1. It should be mentioned that the parameter $\rho_0 \in [0,1)$ plays an crucial role in the adaptive DETS. Different values of the parameter ρ_0 yield to significantly different control performance, for special case, $\rho_0 = 0$, then one has that the next transmission instant $t_{k+1}h = t_k + h$, that is, the event-triggered scheme is regressed to the time-triggered scheme. For the latest transmitted sampled-data $w(\hat{x}_l, t_k h)$, whether or not the new sampled-data $w(\hat{x}_l, \ell_{ki})$ is sent to the controller has much to do with the selected value of the parameter ρ_0. As the work [20], where the parameter ρ_0 is a preset constant, then according to the event-triggered condition (8), the margin of the error $e_t(\hat{x}_l, t) = w(\hat{x}_l, \ell_{ki}) - w(\hat{x}_l, t_k h)$ is determined. The problem is that the sampled data still needs to be updated even when the error is within the allowable range according to event-triggered condition (8) with the preset constant parameter ρ_0. It makes sense to make adaptive adjustment according to the latest transmitted sampled-data $w(\hat{x}_l, t_k h)$ of the system and the new sampled-data $w(\hat{x}_l, \ell_{ki})$, thus, this paper consider the event-triggered scheme with adaptive event-triggered threshold $\rho(\ell_{ki})$ to adapt to the change of the state of the system.

To study the H_∞ control problem of uncertain fuzzy distributed parameter system (4), a point feedback controller based on an adaptive threshold event-triggered sampled data is considered as

Rule j: IF $\theta_1(x,t)$ is χ_1^i, $\theta_2(x,t)$ is χ_2^i, $\cdots$, $\theta_\zeta(x,t)$ is χ_ζ^i, THEN

$$u(x,t) = K_j(\hat{x}_l) w(\hat{x}_l, t_k h), \tag{11}$$

where $j \in \{1, 2, \cdots, r\}$, $x \in [x_{l-1}, x_l)$, $t \in [t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1})$, $w(\hat{x}_l, t_k h)$ is a measurable output; $K_j(\hat{x}_l)$ is the controller gain at position $\hat{x}_l$, which will be designed later.

It is worth noting that factors such as equipment obsolescence, aging and external disturbance may cause actuator execution errors. Based on the work [20], [21], consider the actuator fault in a stochastic process framework as follows:

$$u(x,t,\sigma(t)) = F(\sigma(t)) u(x,t), \tag{12}$$

where $\{\sigma(t), t \geq 0\}$ is a continuous time Markov chain with discrete mode, the number of the modes is finite, and the set of modes is defined as $\mathcal{M} = \{1, 2, 3, \cdots, m\}$, transfer rate matrix $\Lambda = [\pi_{ij}]_{m\times m}$ of Markov chain is governed by

$$\mathcal{P}\{\sigma(t+\lambda) = j \mid \sigma(t) = i\} = \begin{cases} \pi_{ij}\lambda + o(\lambda), \ i \neq j, \\ 1 + \pi_{ii}\lambda + o(\lambda), \ i = j, \end{cases} \tag{13}$$

where $\pi_{ij} \geq 0$, $i \neq j$ is the transfer rate from the mode i at the time of t to the mode j at the time of $t+\lambda$, and $\pi_{ii} = -\sum\limits_{i=1, i\neq j}^{m} \pi_{ij}$; $\lambda > 0$ is an increment in time, $o(\lambda)$ stands for the infinitely small variable with respect to λ, that is, $\lim\limits_{\lambda\to 0} \frac{o(\lambda)}{\lambda} = 0$.

For $\sigma(t) = i_\sigma$, consider the parameters of the actuator $F(i_\sigma)$ satisfying $0 \leq F(i_\sigma) \leq 1$.

Then, a global fuzzy controller is given:

$$u(x,t,\sigma(t)) = F(\sigma(t)) \sum_{j=1}^{r} \mu_j(\theta(x,t)) K_j(\hat{x}_l) w(\hat{x}_l, t_k h), \tag{14}$$

where $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, $x \in [x_{l-1}, x_l)$, $j \in \{1, 2, \cdots, r\}$.

2.2. A closed-loop fuzzy system with communication delay

In order to consider the system (4) and the fuzzy controller (14) based on event-triggered condition in a uniform time frame, similar to the work [22], denote $\mathcal{T}_k = [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, to find out the unified variable about time, the time interval section $\mathcal{T}_k$ is then discussed by case:

(1) If $t_k h + h + \tau_M \geq t_{k+1}h + \tau_{k+1}$, then denote $d(t) = t - t_k h$, $t \in \mathcal{T}_k$;
(2) If $t_k h + h + \tau_M < t_{k+1}h + \tau_{k+1}$, there must exist scalar $m \in \mathbb{N}_+$ such that $t_k h + h + \tau_M < t_{k+1}h + (m+1)h + \tau_M$, the variables $w(\hat{x}_l, t_k h)$ and $w(\hat{x}_l, t_k h + jh)$, $j = 1, 2, \cdots, m$ satisfy the event-triggered control scheme (8), then denote $\tau(t) = t - t_k h - ih$, $t \in \mathcal{T}_k^j$, $j = 0, 1, \cdots, m$, where the time interval $\mathcal{T}_k$ will be divided into $(l+1)$ subintervals $\mathcal{T}_k = \bigcup\limits_{j=0}^{m} \mathcal{T}_k^j$, and

$$\mathcal{T}_k^j = \begin{cases} [t_k h + \tau_k, t_k h + (j+1)h + \tau_M), & j = 0, 1, \cdots, m-1, \\ [t_k h + mh + \tau_k, t_{k+1} h + \tau_{k+1}), & j = m. \end{cases} \tag{15}$$

Based on the above discussion, let $0 \leq \tau_k \leq \tau(t) \leq d$, and $d = h + \tau_M$.

For the case (1), one has

$$e_t(\hat{x}_l, t) = 0. \tag{16}$$

For the case (2), one has

$$\begin{cases} e_t(\hat{x}_l, t) = 0, & t \in \mathcal{T}_k^0, \\ e_t(\hat{x}_l, t) = w(\hat{x}_l, t_k h + jh) - w(\hat{x}_l, t_k h), & t \in \mathcal{T}_k^j, \quad j = 1, 2, \cdots, m. \end{cases} \tag{17}$$

For simplicity, let $\mu_i = \mu_i(\theta(x,t))$, $\mu_{ij} = \mu_i(\theta(x,t))\mu_j(\theta(x,t))$. Combining (4), (11) and (12), the closed-loop fuzzy system can be derived as follows:

$$\begin{aligned} \frac{\partial w(x,t)}{\partial t} &= \frac{\partial}{\partial x}(D(x)\nabla w(x,t)) + \sum_{i=1}^{r} \mu_i (A_i(x) + A_{ui}(x,t)) w(x,t) + \sum_{i=1}^{r} \mu_i C_i(x) \upsilon(x,t) \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_{ij} B_i(x) F(\sigma(t)) K_j [w(x, t-\tau(t)) - e_x(x, t-\tau(t)) - e_t(\hat{x}_l, t)], \end{aligned} \tag{18}$$

where $x \in [x_{l-1}, x_l)$, $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, $l \in \{1, 2, \cdots, \ell\}$, $k \in \{0, 1, 2, \cdots\}$.

In this paper, Lyapunov method and stochastic control method are mainly used to study the feedback H_∞ control problem of distributed parameter system (18) based on event-triggered sampled data. The design of the feedback controller depends on the stability of the system. Therefore, this paper introduces the definition of stochastic exponential stability.

Definition 1. For any initial condition $w_0(x)$, $\sigma(0) \in \mathcal{M}$ and all the permissible uncertainties, if there exists a constant $\zeta > 0$ such that

$$\mathbf{E}\left\{ ||w(x,t)||_{\mathcal{L}_2}^2 \mid w_0(x), \sigma(0) \right\} \leq \zeta e^{-2\beta t} ||w_0(x)||_{\mathcal{L}_2}^2, \tag{19}$$

where $\beta > 0$ is a given scalar, it is said that the closed loop system (18) under the boundary conditions (2), (3) and $\delta(x,t) = 0$ is stochastically exponentially stable (SES) with decay rate β.

Further, for a specified interference attenuation level of $\gamma > 0$, if

$$\mathbf{E}\left\{ \int_0^\infty ||w(x,t)||_{\mathcal{L}_2}^2 dt \right\} < \gamma^2 \int_0^\infty ||\delta(x,t)||_{\mathcal{L}_2}^2 dt, \tag{20}$$

then the closed-loop system (18) under the boundary condition (2) and zero initial condition (3) is said to satisfy the H_∞ performance.

For simplicity, denote $H(t) = \mathbf{E}\left\{\int_0^\infty ||w(x,t)||_{\mathcal{L}_2}^2 dt\right\} - \gamma^2 \int_0^\infty ||\delta(x,t)||_{\mathcal{L}_2}^2 dt$. If condition (19) and $H(t) < 0$ hold, the system (18) satisfies the H_∞ performance with a decay rate β.

3. Stochastically exponential stability and H_∞ performance of the closed-loop systems

The following lemmas are used in the analysis and proof of the stochastic exponential stability of the closed-loop system.

Lemma 1. *[23] If $\frac{dx(\vartheta)}{d\vartheta}$: $[a, b] \to \mathbf{R}^n$ is square integrable, Q is a symmetric positive definite matrix, for any positive value β, then*

$$-\int_a^b e^{2\beta(\vartheta-b)}\frac{dx(\vartheta)}{d\vartheta}^T Q\frac{dx(\vartheta)}{d\vartheta}d\vartheta \le -\frac{2\beta}{1-e^{-2\beta(b-a)}}\xi^T diag\{Q,3Q\}\xi, \tag{21}$$

where

$$\xi = \begin{bmatrix} x(b)-x(a) \\ x(b)+x(a)-\frac{3}{b-a}\int_a^b x(\vartheta)d\vartheta \end{bmatrix}.$$

Lemma 2. *[24] If $M \in \mathbf{R}^{n_1\times n_2}$, $N \in \mathbf{R}^{n_2\times n_3}$ and $F \in \mathbf{R}^{n_2\times n_2}$ are real matrices, and the matrix F satisfies $F^TF \le I$, for any positive number $\epsilon > 0$, then*

$$MFN + (MFN)^T \le \epsilon^{-1}MM^T + \epsilon N^TN.$$

Lemma 3. *[25] For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12}^T \\ * & S_{22}\end{bmatrix}$, where $S_{11} \in \mathbf{R}^{n\times n}$, $S_{12} \in \mathbf{R}^{n\times m}$, $S_{22} \in \mathbf{R}^{m\times m}$, then the following three conditions are equivalent*

(1) $S < 0$;
(2) $S_{11} < 0$, $S_{22} - S_{12}^TS_{11}^{-1}S_{12} < 0$;
(3) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Lemma 4. *[26] Let $0 < \beta_d < 2\beta_0$ and an absolutely continuous scalar function $V : [t_0 - d, \infty) \to [0, \infty)$. If the inequality*

$$\dot V(t) \le -2\beta_0 V(t) + \beta_d \sup_{-d\le\varsigma\le 0} V(t+\varsigma), \quad t \ge t_0, \tag{22}$$

holds, it is then derived that

$$V(t) \le e^{-2\beta(t-t_0)} \sup_{-d\le\varsigma\le 0} V(t+\varsigma), \quad t \ge t_0, \tag{23}$$

where β is the unique solution of

$$\beta = \beta_0 - \frac{\beta_d e^{2d\beta}}{2}. \tag{24}$$

Lemma 5. *[27] Let $x : [a, b] \to \mathbf{R}^n$ be an absolutely continuous function with $\dot x \in L_2(a, b)$ and $x(a) = 0$. Then for any $n \times n$ matrix $Q > 0$,*

$$\int_a^b x^T(s)Qx(s)ds \le \frac{4(b-a)^2}{\pi^2}\int_a^b \dot x^T(s)Q\dot x(s)ds.$$

Lemma 6. *Let $s(t) : \mathbf{R} \to [\alpha_1, \alpha_2]$ is a bounded function about the variable t, where α_1 and α_2 are real numbers; $a_{ij}(s(t))$ is a linear function of the variable s ($i, j = 1, 2, \cdots, n$); $A(s(t)) = [a_{ij}(s(t))] \in \mathbf{R}^{n\times n}$ is a symmetric matrix function. If*

$$A(\alpha_1) \le 0, \quad A(\alpha_2) \le 0,$$

hold, then, for any vector function $\xi(t) : \mathbf{R} \times \mathbf{R}^n \to \mathbf{R}^n$, the matrix function satisfies

$$\xi(t)^T A(s(t))\xi(t) \le 0, \quad s \in [\alpha_1, \alpha_2].$$

Proof. Let $\hat{\xi} = \begin{bmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \cdots & \hat{\xi}_n \end{bmatrix}^T$ is an arbitrary constant vector, one can get

$$\hat{\xi}^T A(s(t))\hat{\xi} = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(s(t))\hat{\xi}_i\hat{\xi}_j \doteq f(s(t)). \tag{25}$$

According to the condition of $A(s(t))$, $f(s)$ is a linear function of the variable s.

According to the monotonicity of linear functions, one has

$$f(\alpha_1) \le f(s) \le f(\alpha_2), \tag{26}$$

or

$$f(\alpha_2) \le f(s) \le f(\alpha_1). \tag{27}$$

If $A(\alpha_1) \le 0$ and $A(\alpha_2) \le 0$, for $s \in [\alpha_1, \alpha_2]$, then $f(s) \le 0$.

Because of the arbitrariness of $\hat{\xi}$, let $\hat{\xi} = \xi(t)$ without loss of generality. For $s \in [\alpha_1, \alpha_2]$, then

$$\xi(t)^T A(s(t))\xi(t) \le 0. \tag{28}$$

This is completed the proof. □

For the stability analysis of the system, choose the following Lyapunov functional

$$V(t,\sigma(t)) = V_1(t,\sigma(t)) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \quad t \in \mathcal{T}_k, \tag{29}$$

where

$$V_1(t,\sigma(t)) = \int_{\underline{x}}^{\overline{x}} \xi_w(x,t)^T P_{\sigma(t)}\xi_w(x,t)dx, \quad \xi_w(x,t) = \begin{bmatrix} w(x,t) \\ \int_{t-\tau(t)}^{t} w(x,\vartheta)d\vartheta \end{bmatrix},$$

$$V_2(t) = \int_{\underline{x}}^{\overline{x}} \int_{t-\tau_M}^{t} e^{2\beta(\vartheta - t)} w(x,\vartheta)^T Q_1 w(x,\vartheta)d\vartheta dx,$$

$$V_3(t) = \int_{\underline{x}}^{\overline{x}} \int_{t-\tau_M}^{t} \int_{\vartheta_2}^{t} e^{2\beta(\vartheta_1 - t)} \frac{\partial w(x,\vartheta_1)^T}{\partial \vartheta_1} Q_2 \frac{\partial w(x,\vartheta_1)}{\partial \vartheta_1} d\vartheta_1 d\vartheta_2 dx,$$

$$V_4(t) = \int_{\underline{x}}^{\overline{x}} \nabla w(x,t)^T D(x) Q_3 \nabla w(x,t)dx,$$

$$V_5(t) = w(\underline{x},t)^T ED(\underline{x})Q_3 w(\underline{x},t),$$

for $\sigma(t) = i_\sigma$, $P_{i_\sigma} = \begin{bmatrix} P_{1,i_\sigma} & P_2^T \\ * & P_3 \end{bmatrix}$, Q_1, Q_2 and Q_3 are symmetric positive definite matrices, and the matrix $D(x)$ and Q_3 are commutative, the matrix $ED(\underline{x})$ and Q_3 are commutative.

Define the following variables:

$$\eta_1(x,t) = \begin{bmatrix} \eta_a^T(x,t) & \eta_b^T(x,t) & \frac{\partial w(x,t)^T}{\partial t} & w(\underline{x},t)^T \end{bmatrix}^T,$$

$$\eta_2(x,t) = \begin{bmatrix} \eta_a^T(x,t) & \eta_b^T(x,t) & \frac{\partial w(x,t)^T}{\partial t} & w(\underline{x},t)^T & \delta(x,t)^T \end{bmatrix}^T,$$

$$\eta_3(x,t) = \begin{bmatrix} \eta_a^T(x,t) & \eta_b^T(x,t) & \frac{\partial w(x,t)^T}{\partial t} & \upsilon(x,t)^T \end{bmatrix}^T,$$

$$\eta_a(x,t) = \begin{bmatrix} w(x,t)^T & w(x,t-\tau(t))^T & w(x,t-\tau_M)^T & e_x(x,t-\tau(t))^T & e_t(\hat{x}_l,t)^T \end{bmatrix}^T,$$

$$\eta_b(x,t) = \begin{bmatrix} \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} w(x,\vartheta)^T d\vartheta & \frac{1}{\tau_M - \tau(t)}\int_{t-\tau_M}^{t-\tau(t)} w(x,\vartheta)^T d\vartheta \end{bmatrix}^T.$$

Theorem 1. *Consider the closed loop system (18) with boundary conditions (2) and (3). For given positive constants τ_M, β, $\hat{\beta}$, the system (18) satisfies the H_∞ performance with a decay rate β if there exist positive constants ϵ_1, ϵ_2, ϵ_3, symmetric positive matrices $P_{1,i_\sigma} \in \mathbf{R}^{n\times n}$ ($i_\sigma \in \mathcal{M}$), $P_3 \in \mathbf{R}^{n\times n}$, $Q_1 \in \mathbf{R}^{n\times n}$, $Q_2 \in \mathbf{R}^{n\times n}$, $Q_3 \in \mathbf{R}^{n\times n}$ and $X \in \mathbf{R}^{n\times n}$, symmetric matrices $Q_4 \in \mathbf{R}^{n\times n}$ and $G \in \mathbf{R}^{2n\times 2n}$, matrix $P_2 \in \mathbf{R}^{n\times n}$ such that*

$$\begin{bmatrix} diag\{Q_2, 3Q_2\} & G^T \\ * & diag\{Q_2, 3Q_2\} \end{bmatrix} \geq 0, \tag{30}$$

$$\begin{bmatrix} P_{1,i_\sigma} - X & P_2^T \\ * & P_3 \end{bmatrix} \geq 0, \tag{31}$$

$$(Q_4^T + P_{1,i_\sigma}))D(x) + D(x)(Q_4 + P_{1,i_\sigma})) - 2\beta Q_3 D(x) \geq 0, \tag{32}$$

$$\Phi(0) \leq 0, \quad \Phi(\tau_M) \leq 0, \tag{33}$$

where β is unique determined by (24), $\vartheta \in [0, \tau_M]$, $\nu = \frac{\overline{x}-\underline{x}}{2\ell}$,

$$\Phi_0(\vartheta) = \{\psi_{ij}\}_{i,j=1}^{9} - \frac{2\beta}{1 - e^{-2\beta\tau_M}}(\Xi_1^T diag\{Q_2, 3Q_2\}\Xi_1 + \Xi_2^T diag\{Q_2, 3Q_2\}\Xi_2 + \Xi_1^T G^T \Xi_2 + \Xi_2^T G \Xi_1),$$

$$\Phi(\vartheta) = \begin{bmatrix} \Phi_0(\vartheta) & \Gamma^T \\ * & -\gamma^2 I \end{bmatrix} - \frac{2\beta}{1 - e^{-2\beta\tau_M}}(\Xi_3^T diag\{Q_2, 3Q_2\}\Xi_3 + \Xi_4^T diag\{Q_2, 3Q_2\}\Xi_4 + \Xi_3^T G^T \Xi_4 + \Xi_4^T G \Xi_3),$$

$$\begin{aligned}
\psi_{11} = {} & (P_{1,i_\sigma} + Q_4^T)A_i(x) + A_i(x)^T(P_{1,i_\sigma} + Q_4) + (P_2 + P_2^T) + \sum_{j_\sigma=1}^{m} \pi_{i_\sigma j_\sigma} P(j_\sigma) \\
& + 2\beta P_{1,i_\sigma} + Q_1 + \tau_M Q_2 + \epsilon_1^{-1} P_{1,i_\sigma} M_i(x) M_i(x)^T P_{1,i_\sigma} \\
& + (\epsilon_1 + \epsilon_2 + \epsilon_3) N_i(x)^T N_i(x) + \epsilon_2^{-1} Q_4^T M_i(x) M_i(x)^T Q_4 \\
& - \frac{\pi^2}{4(\overline{x} - \underline{x})^2}[D(x)(Q_4 + P_{1,i_\sigma} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)D(x)],
\end{aligned}$$

$$\psi_{12} = (P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l), \ \psi_{14} = -(P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l),$$

$$\psi_{15} = -(P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l), \ \psi_{16} = \tau(t)P_3 + 2\beta P_2^T,$$

$$\psi_{19} = \frac{\pi^2}{4(\overline{x} - \underline{x})^2}[D(x)(Q_4 + P_{1,i_\sigma} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)D(x)],$$

$$\psi_{18} = -Q_4^T + A_i(x)^T Q_3, \ \psi_{1,10} = (P_{1,i_\sigma} + Q_4)C_i(x), \ \psi_{22} = -\hat{\beta}X + \rho\Omega(\hat{x}_l),$$

$$\psi_{24} = -\rho\Omega(\hat{x}_l), \ \psi_{25} = -\rho\Omega(\hat{x}_l), \ \psi_{28} = (B_i(x)F(i_\sigma)K_j(\hat{x}_l))^T Q_3, \ \psi_{33} = -e^{-2\beta\tau_M}Q_1,$$

$$\psi_{44} = -\frac{\hat{\beta}\pi^2}{\nu^2}Q_3 D(x) + \rho\Omega(\hat{x}_l), \ \psi_{45} = \rho\Omega(\hat{x}_l), \ \psi_{48} = -(B_i(x)F(i_\sigma)K_j(\hat{x}_l)(\hat{x}_l))^T Q_3,$$

$$\psi_{55} = (\rho - 1)\Omega(\hat{x}_l), \ \psi_{58} = -(B_i(x)F(r_i)K_j(\hat{x}_l)(\hat{x}_l))^T Q_3, \ \psi_{66} = 2\beta\tau_M^2 P_3,$$

$$\psi_{68} = \vartheta P_2, \ \psi_{77} = -\hat{\beta}e^{-2\beta\tau_M}(\tau_M - \vartheta)Q_1,$$

$$\psi_{88} = \tau_M Q_2 - 2Q_3 + \epsilon_3^{-1} Q_3 M_i(x) M_i(x)^T Q_3, \ \psi_{8,10} = Q_3 C_i(x),$$

$$\begin{aligned}
\psi_{99} = {} & -\frac{\pi^2}{4(\overline{x} - \underline{x})^2}[D(x)(Q_4 + P_{1,i_\sigma} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)D(x)] \\
& - [D(\underline{x})E(Q_4 + P_{1,i_\sigma} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)ED(\underline{x})],
\end{aligned}$$

$$\Xi_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 \end{bmatrix}, \ \Xi_2 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 \end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 \end{bmatrix}, \ \Xi_4 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \psi_{1,10}^T & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{8,10}^T & 0 \end{bmatrix}.$$

Proof. Let $\mathfrak{L}$ is the weakly minimal generator operator of stochastic process $\{V(t,\sigma(t))\}$. For each $\sigma(t)=i_\sigma$ ($\sigma\in\mathcal{M}$), define

$$\mathfrak{L}V(t,i_\sigma)=\lim_{\lambda\to 0}\frac{\mathcal{E}\{V(t+\lambda,\sigma(t+\lambda))|\ \sigma(t)=i_\sigma\}-V(t,i_\sigma)}{\lambda}. \tag{34}$$

According to the stochastic stability definition, it is first proved that conditions (30), (31) and (33) can ensure that the closed-loop system (18) under the perturbation function $\delta(x,t)=0$ is stochastically exponentially stable with an attenuation rate β.

Based on Halanay's inequality in Lemma 4, it is derived that

$$\mathfrak{L}V(t,i_\sigma)+2\beta V(t,i_\sigma)-\hat{\beta}\sup_{\vartheta\in[-\tau_M,0]}V(t+\vartheta,i_\sigma)\le 0, \tag{35}$$

then the closed-loop system is stochastically exponentially stable with an attenuation rate β and β is determined by the condition (24).

Calculating the weak minimal generator of Lyapunov functional (29) along the solution of system (18), one has

$$\begin{aligned}\mathfrak{L}V_1(t,i_\sigma)&=2\int_{\underline{x}}^{\overline{x}}\xi_w(x,t)^TP_{1,i_\sigma}\begin{bmatrix}\frac{\partial w(x,t)}{\partial t}\\ w(x,t)\end{bmatrix}dx+\sum_{j_\sigma=1}^{m}\pi_{i_\sigma j_\sigma}w(x,t)^TP_{1,j_\sigma}w(x,t)\\
&=2\int_{\underline{x}}^{\overline{x}}w(x,t)^TP_{1,i_\sigma}\frac{\partial w(x,t)}{\partial t}dx+2\int_{\underline{x}}^{\overline{x}}\int_{t-\tau(t)}^{t}w(x,\vartheta)^Td\vartheta P_2\frac{\partial w(x,t)}{\partial t}dx\\
&\quad+2\int_{\underline{x}}^{\overline{x}}w(x,t)^TP_2^Tw(x,t)dx+2\int_{\underline{x}}^{\overline{x}}\int_{t-\tau(t)}^{t}w(x,\vartheta)^Td\vartheta P_3w(x,t)dx\\
&\quad+\sum_{j_\sigma=1}^{m}\pi_{i_\sigma j_\sigma}w(x,t)^TP_{1,j_\sigma}w(x,t).\end{aligned} \tag{36}$$

$$\mathfrak{L}V_2(t)=-2\beta V_2(t)+\int_{\underline{x}}^{\overline{x}}w(x,t)^TQ_1w(x,t)dx+\int_{\underline{x}}^{\overline{x}}e^{-2\beta\tau_M}w(x,t-\tau_M)^TQ_1w(x,t-\tau_M)dx, \tag{37}$$

$$\mathfrak{L}V_3(t)=-2\beta V_3(t)+\int_{\underline{x}}^{\overline{x}}\tau_Mw(x,t)^TQ_2w(x,t)dx-\int_{\underline{x}}^{\overline{x}}\int_{t-\tau_M}^{t}e^{2\beta(\vartheta_1-t)}w(x,\vartheta_1)^TQ_2w(x,\vartheta_1)d\vartheta_1dx. \tag{38}$$

Combining the boundary conditions (5) and (6) and using integration by parts, one has

$$\begin{aligned}\mathfrak{L}V_4(t)+\mathfrak{L}V_5(t)&=2\int_{\underline{x}}^{\overline{x}}\nabla w(x,t)^TD(x)Q_3\frac{\partial\nabla w(x,t)}{\partial t}dx+2w(\underline{x},t)^TED(\underline{x})Q_3\frac{\partial w(\underline{x},t)}{\partial t}\\
&=2\nabla w(\overline{x},t)^TD(\overline{x})Q_3\frac{\partial w(\overline{x},t)}{\partial t}-2\nabla w(\underline{x},t)^TD(\underline{x})Q_3\frac{\partial w(\underline{x},t)}{\partial t}\\
&\quad-2\int_{\underline{x}}^{\overline{x}}\frac{\partial}{\partial x}(D(x)\nabla w(x,t))Q_3\frac{\partial w(x,t)}{\partial t}dx+2w(\underline{x},t)^TED(\underline{x})Q_3\frac{\partial w(\underline{x},t)}{\partial t}\\
&=-2\int_{\underline{x}}^{\overline{x}}\frac{\partial}{\partial x}(D(x)\nabla w(x,t))Q_3\frac{\partial w(x,t)}{\partial t}dx.\end{aligned} \tag{39}$$

Combining Lemma 4, integration by parts and boundary conditions, one has

$$\begin{aligned}
&2\int_{\underline{x}}^{\overline{x}} w(x,t)^T P_{1,i_\sigma} \frac{\partial}{\partial x}(D(x)\nabla w(x,t))dx 2w(\overline{x},t)^T P_{1,i_\sigma} D(\overline{x})\nabla w(\overline{x},t) \\
&= -2w(\underline{x},t)^T P_{1,i_\sigma} D(\underline{x})\nabla w(\underline{x},t) - 2\int_{\underline{x}}^{\overline{x}} \nabla w(x,t)^T \phi(x) P_{1,i_\sigma} \nabla w(x,t)dx \\
&\le -w(\underline{x},t)^T (P_{1,i_\sigma} D(\underline{x})E + ED(\underline{x})P_{1,i_\sigma})w(\underline{x},t) \\
&\quad - \frac{\pi^2}{2(\overline{x}-\underline{x})^2}\int_{\underline{x}}^{\overline{x}} [w(x,t)-w(\underline{x},t)]^T D(x)P_{1,i_\sigma}[w(x,t)-w(\underline{x},t)]dx. \qquad (40)
\end{aligned}$$

According to Lemma 1 and Lemma 4, it is derived that

$$\begin{aligned}
&-\int_{\underline{x}}^{\overline{x}}\int_{t-\tau_M}^{t} e^{2\beta(\vartheta_1-t)}\frac{\partial w(x,\vartheta_1)^T}{\partial t} Q_2 \frac{\partial w(x,\vartheta_1)}{\partial t} d\vartheta_1 dx \\
&= -\int_{\underline{x}}^{\overline{x}}\int_{t-\tau(t)}^{t} e^{2\beta(\vartheta_1-t)}\frac{\partial w(x,\vartheta_1)^T}{\partial t} Q_2 \frac{\partial w(x,\vartheta_1)}{\partial t} d\vartheta_1 dx - \int_{\underline{x}}^{\overline{x}}\int_{t-\tau_M}^{t-\tau(t)} e^{2\beta(\vartheta_1-t)}\frac{\partial w(x,\vartheta_1)^T}{\partial t} Q_2 \frac{\partial w(x,\vartheta_1)}{\partial t} d\vartheta_1 dx \\
&\le -\frac{2\beta}{1-e^{-2\beta\tau_M}}\left(\frac{1-e^{-2\beta\tau_M}}{1-e^{-2\beta\tau(t)}}\bar{\xi}_a^T(t)\mathrm{diag}\{Q_2,3Q_2\}\bar{\xi}_a(t) + \frac{1-e^{-2\beta\tau_M}}{e^{-2\beta\tau(t)}-e^{-2\beta\tau_M}}\bar{\xi}_b^T(t)\mathrm{diag}\{Q_2,3Q_2\}\bar{\xi}_b(t)\right) \\
&\le -\frac{2\beta}{1-e^{-2\beta\tau_M}}\left(\bar{\xi}_a^T \mathrm{diag}\{Q_2,3Q_2\}\bar{\xi}_a(t)\right) - \frac{2\beta}{1-e^{-2\beta\tau_M}}\left(\bar{\xi}_b^T(t)\mathrm{diag}\{Q_2,3Q_2\}\bar{\xi}_b(t)\right) \\
&\quad - \frac{2\beta}{1-e^{-2\beta\tau_M}}\left(2\bar{\xi}_a^T(t)G\bar{\xi}_b(t)\right), \qquad (41)
\end{aligned}$$

where G is a symmetric matrix with satisfying condition (30) and

$$\bar{\xi}_a(t) = \begin{bmatrix} \int_{\underline{x}}^{\overline{x}}[w(x,t)-w(x,t-\tau(t))]dx \\ \int_{\underline{x}}^{\overline{x}}[w(x,t)+w(x,t-\tau(t)) - \frac{3}{\tau(t)}\int_{t-\tau(t)}^{t} w(x,\vartheta_1)d\vartheta_1]dx \end{bmatrix},$$

$$\bar{\xi}_b(t) = \begin{bmatrix} \int_{\underline{x}}^{\overline{x}}[w(x,t-\tau(t))-w(x,t-\tau_M)]dx \\ \int_{\underline{x}}^{\overline{x}}[w(x,t-\tau(t))+w(x,t-\tau_M) - \frac{3}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)} w(x,\vartheta_1)d\vartheta_1]dx \end{bmatrix}.$$

In view of the closed-loop system (18), for matrices Q_3 and Q_4, one has

$$\begin{aligned}
0 &= [Q_4 w(x,t) + Q_3\frac{\partial w(x,t)}{\partial t}]^T[\frac{\partial}{\partial x}(D(x)\nabla w(x,t)) + \sum_{i=1}^{r}\mu_i(A_i(x)+A_{ui}(x,t))w(x,t) - \frac{\partial w(x,t)}{\partial t} \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_{ij}B_i(x)F(i_\sigma)K_j(\hat{x}_l)(w(x,t-\tau(t)) - e_x(x,t-\tau(t)) - e_t(\hat{x}_l,t))]. \qquad (42)
\end{aligned}$$

Similar to (40), it is obtained that

$$\begin{aligned}
&2\int_{\underline{x}}^{\overline{x}} w(x,t)^T Q_4^T \frac{\partial}{\partial x}(D(x)\nabla w(x,t))dx \le -2w(\underline{x},t)^T Q_4^T D(\underline{x})Ew(\underline{x},t) \\
&\quad - \frac{\pi^2}{2(\overline{x}-\underline{x})^2}\int_{\underline{x}}^{\overline{x}} [w(x,t)-w(\underline{x},t)]^T D(x)Q_4^T[w(x,t)-w(\underline{x},t)]dx. \qquad (43)
\end{aligned}$$

According to Lemma 2, for positive scalars ϵ_1, ϵ_2 and ϵ_3, one has

$$
\begin{aligned}
&2w(x,t)^T P_{1,i_\sigma} A_{ui}(x,t)w(x,t) = 2w(x,t)^T P_{1,i_\sigma} M_i(x)F_i(x,t)N_i(x)w(x,t)\\
&\le \epsilon_1^{-1} w(x,t)^T P_{1,i_\sigma} M_i(x)(P_{1,i_\sigma} M_i(x))^T w(x,t) + \epsilon_1 w(x,t)^T N_i(x)^T N_i(x)w(x,t), \qquad (44)\\
&2w(x,t)^T Q_4^T A_{ui}(x,t)w(x,t) \le \epsilon_2^{-1} w(x,t)^T Q_4^T M_i(x)(Q_4^T M_i(x))^T w(x,t) + \epsilon_2 w(x,t)^T N_i(x)^T N_i(x)w(x,t), \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
2\frac{\partial w(x,t)^T}{\partial t} Q_3 A_{ui}(x,t)w(x,t) &\le \epsilon_3^{-1} \frac{\partial w(x,t)^T}{\partial t} Q_3 M_i(x)(Q_3 M_i(x))^T \frac{\partial w(x,t)}{\partial t}\\
&\quad + \epsilon_3 w(x,t)^T N_i(x)^T N_i(x)w(x,t). \qquad (46)
\end{aligned}
$$

For $x \in [x_{l-1}, x_l]$ and a symmetric positive definite matrix Y, according to Lemma 4, one has

$$
\begin{aligned}
&\int_{x_{l-1}}^{x_l} \nabla w(x,t-\tau(t))^T Y \nabla w(x,t-\tau(t))dx\\
&= \int_{x_{l-1}}^{\hat{x}_l} \nabla w(x,t-\tau(t))^T Y \nabla w(x,t-\tau(t))dx + \int_{\hat{x}_l}^{x_l} \nabla w(x,t-\tau(t))^T Y \nabla w(x,t-\tau(t))dx\\
&\ge \frac{\pi^2}{\Delta_x^2} \int_{x_{l-1}}^{\hat{x}_l} [w(x,t-\tau(t)) - w(\hat{x}_l,t-\tau(t)]^T Y [w(x,t-\tau(t)) - w(\hat{x}_l,t-\tau(t)]dx\\
&\quad + \frac{\pi^2}{\Delta_x^2} \int_{\hat{x}_l}^{x_l} [w(x,t-\tau(t)) - w(\hat{x}_l,t-\tau(t)]^T Y [w(x,t-\tau(t)) - w(\hat{x}_l,t-\tau(t)]dx\\
&= \frac{\pi^2}{\Delta_x^2} \int_{x_{l-1}}^{x_l} e_x(x,t-\tau(t))^T Y e_x(x,t-\tau(t))dx. \qquad (47)
\end{aligned}
$$

It is easily derived that

$$
\begin{aligned}
2\beta V_1(t,i_\sigma) &\le 2\beta w(x,t)^T P_{1,i_\sigma} w(x,t) + 4\beta\tau(t)\left(\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} w(x,\vartheta)^T d\vartheta\right) P_2 w(x,t)\\
&\quad + 2\beta\tau_M^2 \left(\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} w(x,\vartheta)^T d\vartheta\right) P_3 \left(\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} w(x,\vartheta) d\vartheta\right). \qquad (48)
\end{aligned}
$$

Substituting (36)-(48) into (35), one has according to Lemma 5

$$
\begin{aligned}
&\mathfrak{L}V(t,i_\sigma) + 2\beta V(t,i_\sigma) - \hat{\beta} \sup_{\vartheta\in[-\tau_M,0]} V(t+\vartheta,i_\sigma)\\
&< \mathfrak{L}V(t,i_\sigma) + 2\beta V(t,i_\sigma) - \hat{\beta} V_1(t-\tau(t),i_\sigma) - \hat{\beta} V_2(t-\tau(t),i_\sigma) - \hat{\beta} V_4(t-\tau(t),i_\sigma)\\
&< \mathfrak{L}V(t,i_\sigma) + 2\beta V(t,i_\sigma) - \hat{\beta} w(x,t-\tau(t))^T X w(x,t-\tau(t))\\
&\quad - \int_{\underline{x}}^{\overline{x}} \int_{t-\tau_M}^{t-\tau(t)} \hat{\beta} e^{-2\beta\tau_M} w(x,\vartheta)^T Q_1 w(x,\vartheta) d\vartheta dx\\
&\quad - \sum_{l=0}^{\ell} \int_{x_{l-1}}^{x_l} \hat{\beta} \nabla w(x,t-\tau(t))^T D(x) Q_3 \nabla w(x,t-\tau(t))dx
\end{aligned}
$$

$$
\begin{aligned}
&\le \mathfrak{L}V(t,i_\sigma) + 2\beta V(t,i_\sigma) - \hat{\beta} w(x,t-\tau(t))^T X w(x,t-\tau(t)) \\
&\quad - \int_{\underline{x}}^{\overline{x}} \frac{\hat{\beta} e^{-2\beta\tau_M}}{\tau_M - \tau(t)} \int_{t-\tau_M}^{t-\tau(t)} w(x,\vartheta)^T d\vartheta\, Q_1 \int_{t-\tau_M}^{t-\tau(t)} w(x,\vartheta) d\vartheta\, dx \\
&\quad - \sum_{l=0}^{\ell} \int_{x_{l-1}}^{x_l} \hat{\beta} \nabla w(x,t-\tau(t))^T D(x) Q_3 \nabla w(x,t-\tau(t)) dx \\
&= \sum_{l=0}^{\ell} \int_{x_{l-1}}^{x_l} \eta_1(t)^T \Phi_0(\tau(t)) \eta_1(t) dx,
\end{aligned} \tag{49}
$$

where the matrix X satisfies $\begin{bmatrix} P_{1,i_\sigma} - X & P_2^T \\ * & P_3 \end{bmatrix} \ge 0$.

Combining conditions (33) and Lemma 3, it can be derived that

$$\Phi_0(0) + \gamma^{-2}\Gamma^T\Gamma \le 0, \tag{50}$$

$$\Phi_0(\tau_M) + \gamma^{-2}\Gamma^T\Gamma \le 0. \tag{51}$$

According to Lemma 6, it can be obtained that $\Phi_0(\tau(t)) \le 0$, which implies that

$$\mathfrak{L}V(t,i_\sigma) - \hat{\beta} \sup_{\vartheta\in[-\tau_M,0]} V(t+\vartheta,i_\sigma) < 0. \tag{52}$$

Therefore, the closed-loop system (18) is stochastically exponentially stable with a decay rate of β under the condition that the disturbance function $\delta(x,t) = 0$.

In the following, it will be shown that the closed-loop system (18) in a disturbed environment satisfies the H_∞ performance with a decay rate of β.

Similar to the above analysis, in the presence of disturbances, one has

$$
\begin{aligned}
&\mathfrak{L}V(t,i_\sigma) + 2\beta V(t,i_\sigma) - \hat{\beta} \sup_{\vartheta\in[-\tau_M,0]} V(t+\vartheta,i_\sigma) - \int_{\underline{x}}^{\overline{x}} \gamma^2 \delta(x,t)^T \delta(x,t) dx \\
&\le \eta_2(t)^T \Phi(\tau(t)) \eta_2(t) \le 0.
\end{aligned} \tag{53}
$$

Therefore, the closed-loop system (18) can reach the H_∞ performance with a decay rate β. This completes the proof. □

Remark 2. It is worth mentioning that $\nabla w(x,t) = \nabla[w(x,t) - w(\underline{x},t)]$ is considered in the inequality (40). If one considers $\nabla w(x,t) = \nabla[w(x,t) - w(\overline{x},t)]$, where the boundary condition is $w(\overline{x},t) = 0$. According to integration by parts, one has

$$
\begin{aligned}
&2\int_{\underline{x}}^{\overline{x}} w(x,t)^T P_{1,i_\sigma} \frac{\partial}{\partial x}(D(x)\nabla w(x,t)) dx \\
&\le -w(\underline{x},t)^T (P_{1,i_\sigma} D(\underline{x}) E + E D(\underline{x}) P_{1,i_\sigma}) w(\underline{x},t) - \frac{\pi^2}{2(\overline{x}-\underline{x})^2} \int_{\underline{x}}^{\overline{x}} w(x,t)^T D(x) P_{1,i_\sigma} w(x,t) dx.
\end{aligned} \tag{54}
$$

Theorem 2. *Given positive numbers τ_M, β, $\hat{\beta}$, the closed-loop system (18) satisfies the H_∞ performance with an attenuation rate β if there are real numbers ϵ_1, ϵ_2, ϵ_3, symmetric positive matrices $P_{i_\sigma} \in \mathbf{R}^{n\times n}$ ($i_\sigma \in \mathcal{M}$), $P_3 \in \mathbf{R}^{n\times n}$, $Q_1 \in \mathbf{R}^{n\times n}$, $Q_2 \in \mathbf{R}^{n\times n}$, $Q_3 \in \mathbf{R}^{n\times n}$ and $X \in \mathbf{R}^{n\times n}$, the symmetric matrices $Q_4 \in \mathbf{R}^{n\times n}$, $G \in \mathbf{R}^{2n\times 2n}$, and matrix $P_2 \in \mathbf{R}^{n\times n}$ such that*

$$\begin{bmatrix} diag\{Q_2, 3Q_2\} & G^T \\ * & diag\{Q_2, 3Q_2\} \end{bmatrix} \geq 0, \tag{55}$$

$$\begin{bmatrix} P_{1,i_\sigma} - X & P_2^T \\ * & P_3 \end{bmatrix} \geq 0, \tag{56}$$

$$(Q_4^T + P_{1,i_\sigma}))D(x) + D(x)(Q_4 + P_{1,i_\sigma})) - 2\beta Q_3 D(x) \geq 0, \tag{57}$$

$$(Q_4^T + P_{1,i_\sigma}))D(\underline{x})E + ED(\underline{x})(Q_4 + P_{1,i_\sigma})) - 2\beta Q_3 D(\underline{x})E \geq 0, \tag{58}$$

$$\hat{\Phi}(0) \leq 0, \quad \hat{\Phi}(\tau_M) \leq 0, \tag{59}$$

where β is uniquely determined by (24), $\vartheta \in [0, \tau_M]$, $\nu = \frac{\overline{x}-\underline{x}}{2\ell}$,

$$\hat{\Phi}_0(\vartheta) = \{\hat{\psi}_{ij}\}_{i,j=1}^{8} - \frac{2\beta}{1 - e^{-2\beta\tau_M}}(\hat{\Xi}_1^T diag\{Q_2, 3Q_2\}\hat{\Xi}_1 + \hat{\Xi}_2^T diag\{Q_2, 3Q_2\}\hat{\Xi}_2 + \hat{\Xi}_1^T G^T \hat{\Xi}_2 + \hat{\Xi}_2^T G \hat{\Xi}_1),$$

$$\begin{aligned} \hat{\Phi}(\vartheta) = {} & \begin{bmatrix} \hat{\Phi}_0(\vartheta) & \Gamma^T \\ * & -\gamma^2 I \end{bmatrix} - \frac{2\beta}{1 - e^{-2\beta\tau_M}}(\Xi_3^T diag\{Q_2, 3Q_2\}\Xi_3 + \Xi_4^T diag\{Q_2, 3Q_2\}\Xi_4 \\ & + \Xi_3^T G^T \Xi_4 + \Xi_4^T G \Xi_3), \end{aligned} \tag{60}$$

$$\begin{aligned} \hat{\psi}_{11} = {} & (P_{1,i_\sigma} + Q_4^T)A_i(x) + A_i(x)^T(P_{1,i_\sigma} + Q_4) + (P_2 + P_2^T) + \sum_{j_\sigma=1}^{m} \pi_{i_\sigma j_\sigma} P_{1,j_\sigma} + 2\beta P_{1,i_\sigma} + Q_1 + \tau_M Q_2 \\ & + \epsilon_1^{-1} P_{1,i_\sigma} M_i(x) M_i(x)^T P_{1,i_\sigma} + (\epsilon_1 + \epsilon_2 + \epsilon_3) N_i(x)^T N_i(x) + \epsilon_2^{-1} Q_4^T M_i(x) M_i(x)^T Q_4 \\ & - \frac{\pi^2}{4(\overline{x} - \underline{x})^2}[D(x)(Q_4 + P_{1,r_i} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)D(x)], \end{aligned}$$

$\hat{\psi}_{12} = (P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l)$, $\hat{\psi}_{14} = -(P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l)$,
$\hat{\psi}_{15} = -(P_{1,i_\sigma} + Q_4^T)B_i(x)F(i_\sigma)K_j(\hat{x}_l)$, $\hat{\psi}_{16} = \tau(t)P_3 + 2\beta P_2^T$,
$\hat{\psi}_{18} = -Q_4^T + A_i(x)^T Q_3$, $\hat{\psi}_{1,9} = (P_{1,i_\sigma} + Q_4)C_i(x)$, $\hat{\psi}_{22} = -\hat{\beta}X + \rho\Omega(\hat{x}_l)$,
$\hat{\psi}_{24} = -\rho\Omega(\hat{x}_l)$, $\hat{\psi}_{25} = -\rho\Omega(\hat{x}_l)$, $\hat{\psi}_{28} = (B_i(x)F(i_\sigma)K_j(\hat{x}_l))^T Q_3$, $\hat{\psi}_{33} = -e^{-2\beta\tau_M} Q_1$,
$\hat{\psi}_{44} = -\frac{\hat{\beta}\pi^2}{\nu^2} Q_3\phi(x) + \rho\Omega(\hat{x}_l)$, $\hat{\psi}_{45} = \rho\Omega(\hat{x}_l)$, $\hat{\psi}_{48} = -(B_i(x)F(i_\sigma)K_j(\hat{x}_l))^T Q_3$,
$\hat{\psi}_{55} = (\rho - 1)\Omega(\hat{x}_l)$, $\hat{\psi}_{58} = -(B_i(x)F(i_\sigma)K_j(\hat{x}_l))^T Q_3$, $\hat{\psi}_{66} = 2\beta\tau_M^2 P_3$,
$\hat{\psi}_{68} = \vartheta P_2$, $\hat{\psi}_{77} = -\hat{\beta}e^{-2\beta\tau_M}(\tau_M - \vartheta)Q_1$,
$\hat{\psi}_{88} = \tau_M Q_2 - 2Q_3 + \epsilon_3^{-1} Q_3 M_i(x) M_i(x)^T Q_3$, $\hat{\psi}_{8,9} = Q_3 C_i(x)$,

$$\hat{\Xi}_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 \end{bmatrix}, \ \hat{\Xi}_2 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 \end{bmatrix},$$

$$\hat{\Xi}_3 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 \end{bmatrix}, \ \hat{\Xi}_4 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 \end{bmatrix},$$

$$\hat{\Gamma} = \begin{bmatrix} \psi_{1,9}^T & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{8,9}^T \end{bmatrix}.$$

Proof. Combining inequality (54) and the vector $\eta_3(t)$, similar to the proof of Theorem 1, it can be obtained that

$$\begin{aligned} & \mathfrak{L}V(t, i_\sigma) + 2\beta V(t, i_\sigma) - \hat{\beta} \sup_{\vartheta \in [-\tau_M, 0]} V(t + \vartheta, i_\sigma) - \int_{\underline{x}}^{\overline{x}} \gamma^2 \delta(x, t)^T \delta(x, t) dx \\ & \leq \eta_3(t)^T \hat{\Phi}(\tau(t)) \eta_3(t) \\ & \leq 0, \end{aligned} \tag{61}$$

where $\hat{\Phi}(\tau(t))$ equals the value when $\vartheta = \tau(t)$ for $\hat{\Phi}(\vartheta)$ in the condition (60). □

4. Design of the control gain

In this section, parameters of the controller will be designed based on the H_∞ stability criteria given in Theorems 1 and 2. To solve the problems with using the LMI tool in Matlab, we deal with nonlinear terms $\epsilon_1^{-1} P_{1,i_\sigma} M_i(x) M_i(x)^T P_{1,i_\sigma}$, $\epsilon_2^{-1} Q_4 M_i(x) M_i(x)^T Q_4$ and $\epsilon_3^{-1} Q_3 M_i(x) M_i(x)^T Q_3$ by Lemma 3. Thus, equivalent results of Theorems 1 and 2 are obtained in the following theorems.

Theorem 3. *For positive constants τ_M, β, $\hat{\beta}$, closed loop system (18) with the boundary conditions (2) and (3) satisfies the H_∞ performance if there exist positive constants ϵ_1, ϵ_2, ϵ_3, symmetric positive matrices $P_{1,i_\sigma} \in \mathbf{R}^{n\times n}$ ($i_\sigma \in \mathcal{M}$), $P_3 \in \mathbf{R}^{n\times n}$, $Q_1 \in \mathbf{R}^{n\times n}$, $Q_2 \in \mathbf{R}^{n\times n}$, $Q_3 \in \mathbf{R}^{n\times n}$ and $X \in \mathbf{R}^{n\times n}$, symmetric matrices $Q_4 \in \mathbf{R}^{n\times n}$ and $G \in \mathbf{R}^{2n\times 2n}$, matrix $P_2 \in \mathbf{R}^{n\times n}$ such that (30)-(32) and*

$$\Phi_1^c(0) \le 0, \qquad \Phi_1^c(\tau_M) \le 0 \tag{62}$$

hold, where $\Upsilon_1 = \begin{bmatrix} P_{1,i_\sigma} M_i(x) \\ \mathbf{0}_{9n\times n} \end{bmatrix}$, $\Upsilon_2 = \begin{bmatrix} Q_4^T M_i(x) \\ \mathbf{0}_{9n\times n} \end{bmatrix}$, $\Upsilon_3 = \begin{bmatrix} \mathbf{0}_{9n\times n} \\ Q_3 M_i(x) \end{bmatrix}$, $\Phi_1(\vartheta) = \begin{bmatrix} \Phi_0^c(\vartheta) & \Upsilon_1^T & \Upsilon_2^T & \Upsilon_3^T \\ * & \epsilon_1 I & 0 & 0 \\ * & * & \epsilon_2 I & 0 \\ * & * & * & \epsilon_3 I \end{bmatrix}$,

$$\Phi_0^c(\vartheta) = \{\psi_{ij}^c\}_{i,j=1}^9 - \frac{2\beta}{1-e^{-2\beta\tau_M}}(\Xi_5^T diag\{Q_2, 3Q_2\}\Xi_5 + \Xi_6^T diag\{Q_2, 3Q_2\}\Xi_6 + \Xi_5^T G^T \Xi_6 + \Xi_6^T G \Xi_5),$$

$$\Phi^c(\vartheta) = \begin{bmatrix} \Phi_1(\vartheta) & \Gamma_c^T \\ * & -\gamma^2 I \end{bmatrix} - \frac{2\beta}{1-e^{-2\beta\tau_M}}(\Xi_7^T diag\{Q_2, 3Q_2\}\Xi_7$$
$$+ \Xi_8^T diag\{Q_2, 3Q_2\}\Xi_8 + \Xi_7^T G^T \Xi_8 + \Xi_8^T G \Xi_7),$$

$$\Xi_5 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_6 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_7 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_8 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_c = \begin{bmatrix} \psi_{1,10}^T & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{8,10}^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

and $\{\psi_{ij}^c\}_{i,j=1}^9$ is from $\{\psi_{ij}\}_{i,j=1}^9$ in (33) with ψ_{11} and ψ_{88} being instead by ψ_{11}^c and ψ_{88}^c, respectively:

$$\psi_{11}^c = (P_{1,i_\sigma} + Q_4^T)A_i(x) + A_i(x)^T(P_{1,i_\sigma} + Q_4) + (P_2 + P_2^T) + \sum_{j_\sigma=1}^m \pi_{i_\sigma j_\sigma} P_{1,j_\sigma}$$
$$+ 2\beta P_{1,i_\sigma} + Q_1 + \tau_M Q_2 + (\epsilon_1 + \epsilon_2 + \epsilon_3)N_i(x)^T N_i(x)$$
$$- \frac{\pi^2}{4(\overline{x} - \underline{x})^2}[D(x)(Q_4 + P_{1,i_\sigma} - \beta Q_3) + (Q_4^T + P_{1,i_\sigma} - \beta Q_3)D(x)],$$
$$\psi_{88}^c = \tau_M Q_2 - 2Q_3.$$

Theorem 4. *Given positive numbers τ_M, β, $\hat{\beta}$, the closed-loop system (18) satisfies the H_∞ performance with an attenuation rate β if there are real numbers ϵ_1, ϵ_2, ϵ_3, symmetric positive matrices $P_{i_\sigma} \in \mathbf{R}^{n\times n}$ ($i_\sigma \in \mathcal{M}$), $P_3 \in \mathbf{R}^{n\times n}$, $Q_1 \in \mathbf{R}^{n\times n}$, $Q_2 \in \mathbf{R}^{n\times n}$, $Q_3 \in \mathbf{R}^{n\times n}$ and $X \in \mathbf{R}^{n\times n}$, the symmetric matrices $Q_4 \in \mathbf{R}^{n\times n}$, $G \in \mathbf{R}^{2n\times 2n}$, and matrix $P_2 \in \mathbf{R}^{n\times n}$ such that conditions (55)-(59) and*

$$\hat{\Phi}(0) \le 0, \qquad \hat{\Phi}(\tau_M) \le 0 \tag{63}$$

hold, where $\Upsilon_4 = \begin{bmatrix} P_{1,i_\sigma} M_i(x) \\ \mathbf{0}_{8n\times n} \end{bmatrix}$, $\Upsilon_5 = \begin{bmatrix} Q_4^T M_i(x) \\ \mathbf{0}_{8n\times n} \end{bmatrix}$, $\Upsilon_6 = \begin{bmatrix} \mathbf{0}_{8n\times n} \\ Q_3 M_i(x) \end{bmatrix}$, $\hat{\Phi}_1^c(\vartheta) = \begin{bmatrix} \hat{\Phi}_0^c(\vartheta) & \Upsilon_4^T & \Upsilon_5^T & \Upsilon_6^T \\ * & \epsilon_1 I & 0 & 0 \\ * & * & \epsilon_2 I & 0 \\ * & * & * & \epsilon_3 I \end{bmatrix}$,

$$\begin{aligned}
\hat{\Phi}_0^c(\vartheta) &= \{\hat{\psi}_{ij}^c\}_{i,j=1}^8 \\
&\quad - \frac{2\beta}{1-e^{-2\beta\tau_M}}\Big((\hat{\Xi}_5^c)^T diag\{Q_2, 3Q_2\}\hat{\Xi}_5^c + (\hat{\Xi}_6^c)^T diag\{Q_2, 3Q_2\}\hat{\Xi}_6^c + (\hat{\Xi}_5^c)^T G^T \hat{\Xi}_6^c + (\hat{\Xi}_6^c)^T G \hat{\Xi}_{5^c}\Big), \\
\hat{\Phi}^c(\vartheta) &= \begin{bmatrix} \hat{\Phi}_1^c(\vartheta) & \Gamma^T \\ * & -\gamma^2 I \end{bmatrix} \\
&\quad - \frac{2\beta}{1-e^{-2\beta\tau_M}}\Big((\Xi_7^c)^T diag\{Q_2, 3Q_2\}\Xi_7^c + (\Xi_8^c)^T diag\{Q_2, 3Q_2\}\Xi_8^c + (\Xi_7^c)^T G^T \Xi_8^c + (\Xi_8^c)^T G \Xi_7^c\Big), \\
\hat{\Xi}_5^c &= \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \hat{\Xi}_6^c = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\hat{\Xi}_7^c &= \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\hat{\Xi}_8^c &= \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & 0 & -3I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\hat{\Gamma} &= \begin{bmatrix} \psi_{1,9}^T & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{8,9}^T & 0 & 0 & 0 \end{bmatrix},
\end{aligned}$$

and $\{\hat{\psi}_{ij}^c\}_{i,j=1}^9$ *is from* $\{\hat{\psi}_{ij}\}_{i,j=1}^9$ *in (59) with* $\hat{\psi}_{11}$ *and* $\hat{\psi}_{88}$ *being instead by* ψ_{11}^c *and* ψ_{88}^c *of Theorem 3.*

Remark 3. Notice that conditions (62) and (63) are bilinear because parameter $K_j(\hat{x}_l)$ is coupled with parameters P_{1,i_σ}, Q_3 and Q_4, respectively. To solve this problem, we using the following algorithm:

- Step 1, choose τ_M, β, $\hat{\beta}$, $\bar{P}_{1,i_\sigma}$, $\bar{Q}_3$ and $\bar{Q}_4$; set $P_{1,i_\sigma} = \bar{P}_{1,i_\sigma}$, $Q_3 = \bar{Q}_3$ and $Q_4 = \bar{Q}_4$;
- Step 2, if there is no feasible solution for (30)-(32) and (62), then reset parameters $\bar{P}_{1,i_\sigma}$, $\bar{Q}_3$ and $\bar{Q}_4$; otherwise, output P_{1,i_σ}, Q_3 and Q_4;
- Step 3, pick up τ_M, β, $\hat{\beta}$, P_{1,i_σ}, Q_3 and Q_4; output $K_j(\hat{x}_l)$ with solving (30)-(32) and (62).

The controller gain of Theorem 4 is also obtained by using this algorithm.

Remark 4. To get $\bar{P}_{1,i_\sigma}$, $\bar{Q}_3$ and $\bar{Q}_4$ in Step 1 of Remark 3, one can choose a suitable initial value $\bar{K}_j(\hat{x}_l)$ by referencing the literature and set $K_j(\hat{x}_l) = \bar{K}_j(\hat{x}_l)$, then, $\bar{P}_{1,i_\sigma}$, $\bar{Q}_3$ and $\bar{Q}_4$ can be obtained by solving LMIs in Theorem 3 or Theorem 4. In practice, they can also be chosen by experience or referencing the literature.

5. Numerical simulations

Two examples are given to illustrate the effectiveness of the proposed controller and control method.

5.1. A numerical example

Consider one-dimensional T-S fuzzy distribution parameter system with uncertain parameters

$$\begin{aligned}
\frac{\partial w(x,t)}{\partial t} &= \frac{\partial}{\partial x}(D(x)\nabla w(x,t)) + \sum_{i=1}^{2}\mu_i(\theta(x,t))(A_i(x) + A_{ui}(x,t))w(x,t) \\
&\quad + \sum_{i=1}^{2}\mu_i(\theta(x,t))B_i(x)u(x,t) + \sum_{i=1}^{2}\mu_i(\theta(x,t))C_i(x)\upsilon(x,t), \qquad (64)
\end{aligned}$$

the boundary condition is

$$\nabla w(0,t) = w(0,t), \quad w(\pi,t) = 0, \quad t \geq 0, \tag{65}$$

the initial condition is

$$w_0(x) = e^{x-\pi} - e^{\sin(x)}, \quad x \in [\underline{x}, \overline{x}] = [0,\pi], \tag{66}$$

where

$$\begin{aligned}
&\mu_1(\theta(x,t)) = \sin(w(x,t))^2, \quad A_1(x) = \sin(x), \quad B_1(x) = 1+0.1x, \quad C_1(x) = \sin(x),\\
&F_1 = 0.5, \quad M_1 = \sin(x), \quad N_1 = \sin(x), \quad A_{u1}(x,t) = 0.5\sin(x)\sin(xt)\sin(x),\\
&\mu_2(\theta(x,t)) = \cos(w(x,t))^2, \quad A_2(x) = \sin(2x), \quad B_2(x) = 1+0.2x, \quad C_2(x) = \cos(2x),\\
&F_2 = 1, \quad M_2 = \cos(x), \quad N_2 = \cos(x), \quad A_{u2}(x,t) = \cos(x)\cos(xt)\cos(x),\\
&D(x) = 1+0.1\cos(x), \quad \upsilon(x,t) = \frac{\sin(xt)}{1+x^2}.
\end{aligned}$$

In this section, consider $\rho_0 = 0.01$, $h = 0.1$, the set of finite modes of Markov chain $\{\sigma(t), t \geq 0\}$ is $\mathcal{M} = \{1,2\}$, and the state transfer rate matrix is

$$\Gamma_p = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}. \tag{67}$$

The position intervals are divided into two parts, namely

$$[0,\pi] = \cup_{l=1}^{2}[\frac{5(l-1)\pi}{10}, \frac{5l\pi}{10}], \quad l = 1,2, \quad \hat{x}_1 = \frac{\pi}{4}, \quad \hat{x}_2 = \frac{3\pi}{4}. \tag{68}$$

According to Theorem 1, one can derive $\gamma = 10.0024$, and the controller gain are

$$K_1(\frac{\pi}{4}) = -4.2508, \quad K_1(\frac{3\pi}{4}) = -4.2482, \tag{69}$$

$$K_2(\frac{\pi}{4}) = -4.2112, \quad K_2(\frac{3\pi}{4}) = -4.2641. \tag{70}$$

The left one of Fig. 1 shows the spatiotemporal evolution of the system (64) state without input control. According to the left one of Fig. 1, it can be seen that the state of the system (64) is unstable without input control.

The right one of Fig. 1 shows the spatiotemporal evolution of the state of system (64) under the proposed event-triggered controller. Furthermore, one can obtain that $H(t) = \mathbf{E}\left\{\int_0^{10} ||w(x,t)||^2_{\mathcal{L}_2} dt\right\} - \gamma^2 \int_0^{\infty} ||\delta(x,t)||^2_{\mathcal{L}_2} dt$. If $H(t) < 0$, which means that (20) holds, which indicates that the H_∞ control performance in (20) is ensured. It can be easily found from the spatiotemporal evolution that the system (64) under the controller of event triggered sampling data can quickly reach convergence. In addition, as shown in left one of Fig. 2, the evolution of the variable $H(t)$ is given, indicating that the system (64) satisfies the H_∞ performance.

As shown in the middle one of Fig. 2, the stochastic response of Markov jump mode $\sigma(t)$ is given. According to event-triggered conditions (10), one can derive event-triggered instant $t_k h$ and the trigger time interval $t_{k+1}h + \tau_{k+1} - t_k h - \tau_k$, $k \in \mathbb{N}$. Compared with the traditional periodic sampling control, the control method based on event-triggered control scheme can significantly reduce the unnecessary sampling data transmission and controller update frequency. As shown in the right one of Fig. 2, the event triggered instant is shown on the horizontal axis, and the ordinate shows the trigger time interval. As shown in Fig. 3, the response of the control input at the first spatial sampling location of $x = 0.25\pi$ and the second spatial sampling location of $x = 0.75\pi$ are given, respectively.

Remark 5. Choose

$$P_{1,1} = 0.1959, \quad P_{1,2} = 0.1961, \quad Q_3 = 0.1961, \quad Q_4 = 0.1961. \tag{71}$$

According to the results of literature [28,29] and this chapter, under the initial threshold of $\rho_0 = 0.1,\ 0.3,\ 0.5,\ 0.7$ in the event trigger condition, the maximum delay τ_M allowed by the system (64) is calculated respectively.

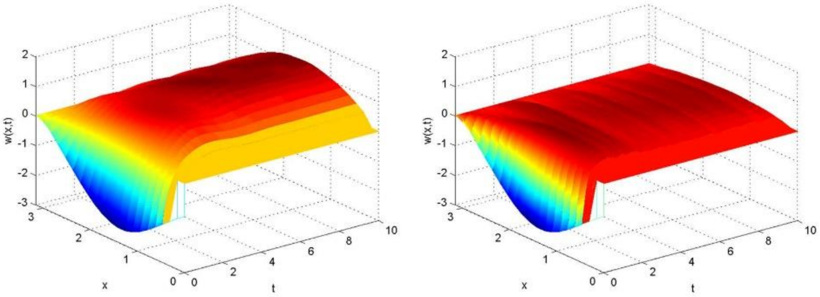


Fig. 1. The left figure shows the spatio-temporal evolution of the system (64) without input control; the right one shows the spatio-temporal evolution of the system (64) with event-triggered control.

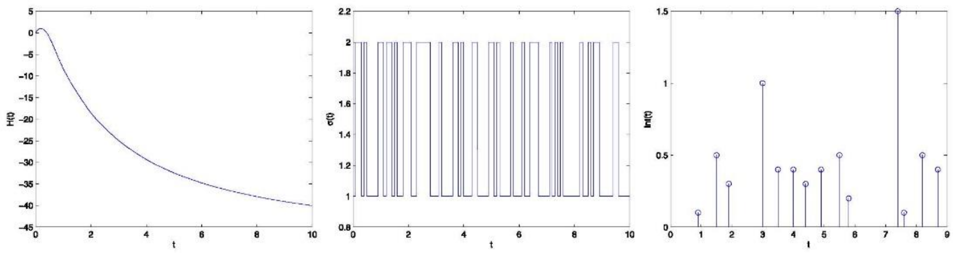


Fig. 2. The left figure shows the evolution of the variable $H(t) = \mathbf{E}\left\{\int_0^{10} ||w(x,t)||^2_{\mathcal{L}_2} dt\right\} - \gamma^2 \int_0^{10} ||\delta(x,t)||^2_{\mathcal{L}_2} dt$; the middle one shows the switching trajectory of the Markov jump mode; the right one shows the release instants and release intervals $int(t) = t_{k+1}h + \tau_{k+1} - t_k h - \tau_k$.

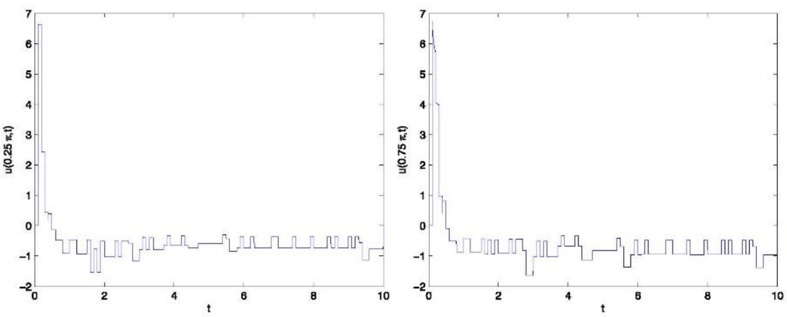


Fig. 3. The left figure shows the trajectory of the input at $x = 0.25\pi$; the right one shows the trajectory of the input at $x = 0.75\pi$.

As shown in Table 1, the admissible delay cannot be obtained by using the results of the work [28], which indicates that the results of the work [28] are conservative. The admissible delay can be obtained by using the results of the work [29], which indicates that the results of the work [29] are less conservative, but the maximum delay is smaller. The allowable delay obtained from the results of this chapter is larger, which indicates that the results of this paper are less conservative.

As shown in Table 2, one can derive the number of the sampled-data received by the controller under different sampling periods, where the number of the sampled-data received by using the periodic control method is 1000,

Table 1
Calculate the maximum allowable delay τ_M.

Different methods	$\rho_0 = 0.1$	$\rho_0 = 0.3$	$\rho_0 = 0.5$	$\rho_0 = 0.7$
[28]	NaN	NaN	NaN	NaN
[29]	0.43	0.40	0.39	0.38
Theorem 1	0.71	0.64	0.60	0.59
Theorem 2	0.71	0.64	0.60	0.59

Table 2
The amount of sampled-data received by the controller within $t \in [0, 10]$ is calculated under different sampling periods h.

Different methods	$h = 0.01$	$h = 0.03$	$h = 0.05$	$h = 0.07$
The periodic control method	1000	1000	1000	1000
Event-triggered control scheme with constant threshold	74	46	43	36
Event-triggered control scheme with adaptive threshold	64	43	37	34

Table 3
Calculate the maximum allowable delay τ_M using Theorem 1.

(F_1, F_2)	(0.5, 1)	(0.5, 0.8)	(0.3, 0.8)	(0.3, 0.5)
τ_M	0.71	0.70	0.64	0.53

Table 4
Calculate the amount of sampled-data received by the controller within $t \in [0, 10]$.

(F_1, F_2)	(0.5, 1)	(0.5, 0.8)	(0.3, 0.8)	(0.3, 0.5)
The amount of triggering event	64	68	75	79

the number of the sampled-data received by using the control method based on event-triggered control scheme with fixed threshold is significantly reduced. It can be seen obviously that the control method based on event-triggered control scheme with adaptive threshold receives the minimum number of sampled data. Therefore, the event-triggered controller based on adaptive threshold proposed in this paper can minimize unnecessary data transmission of sampling and controller update frequency. Thus, resources can be saved more effectively and the service life of the controller can be extended.

Remark 6. In the following, we discuss the impacts of actuator faults in the following two aspects: a) from Table 3, the maximum allowable delay τ_M is smaller as parameters (F_1, F_2) decrease. b) from Table 4, the amount of sampled-data received by the controller is larger as parameters (F_1, F_2) decrease.

5.2. A catalytic rod

The event-generator-based T-S fuzzy control of the temperature profile of a catalytic rod is discussed. The dynamic model with exogenous disturbance $\upsilon(x, t)$ is given in [36] as follows:

$$\frac{\partial w(x,t)}{\partial t} = d\frac{\partial^2 w(x,t)}{\partial x^2} + a(x)\big(e^{-\frac{b}{1+w(x,t)}} - e^{-b}\big) + c\big(\bar{f}(x)u(x,t) - w(x,t)\big) + \upsilon(x,t), \tag{72}$$

the initial condition is $w(x, 0) = w_0(x)$ and the boundary conditions are $\nabla w(0, t) = 0.5w(0, t)$ and $w(\pi, t) = 0$, where $w(x, t)$, $a(x)$, c, d, b and $u(x, t)$ are the temperature in the reactor, heat of reaction, heat transfer coefficient, dimensionless diffusion coefficient, activation energy, manipulated input, respectively; $f(x) = c\bar{f}(x)$ is an actuator distribution function.

Parameters of the system are given for $x \in [0, \pi]$: $d = 1$, $a(x) = 60 - 6e^{-0.1x}$, $b = 4$, $c = 2$, $\bar{f}(x) = 0.1\cos(0.1x)$, the initial condition $w_0(x) = 1.7\sin(x)$ and $\upsilon(x, t) = \frac{\cos(xt)}{1+t}$. The open-loop evolution profile of rod temperature is shown in the left one of Fig. 4.

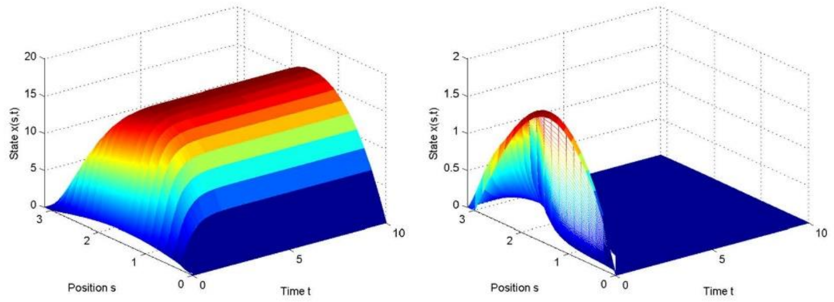


Fig. 4. The left figure shows the spatio-temporal evolution of the system (72) without input control; the right one shows the spatio-temporal evolution of the system (73) with event-triggered control.

Following [36], the system can be exactly represented by the following T-S fuzzy system.

Rule 1: IF $w(x,t)$ is not "about 1.7731", THEN

$$\frac{\partial w(x,t)}{\partial t}=\frac{\partial^2 w(x,t)}{\partial x^2}+A_1(x)w(x,t)+B_1(x)u(x,t)+\upsilon(x,t);$$

Rule 2: IF $w(x,t)$ is "about 1.7731", THEN

$$\frac{\partial w(x,t)}{\partial t}=\frac{\partial^2 w(x,t)}{\partial x^2}+A_1(x)w(x,t)+B_2(x)u(x,t)+\upsilon(x,t),$$

where $A_1(x)=-c$, $A_2(x)=0.1353a(x)-c$, $B_1(x)=B_2(x)=c\bar{f}(x)$, and membership functions are given as follows:

$$\mu_2(w(x,t))=1-\mu_1(w(x,t)),\quad \mu_1(w(x,t))=\begin{cases}\frac{0.1353w(x,t)-(e^{\frac{-b}{1+w(x,t)}}-e^{-b})}{0.1353w(x,t)}, & w(x,t)\neq 0\\ 0.4582, \quad w(x,t)=0.\end{cases}$$

By using a standard inference method, the following form of weighted average of the local models is given

$$\frac{\partial w(x,t)}{\partial t}=\frac{\partial^2 w(x,t)}{\partial x^2}+\sum_{i=1}^{2}\mu_i(w(x,t))A_i(x)w(x,t)+B(x)u(x,t)+\upsilon(x,t).$$

Considering the actual situation may be that the measuring tool is not accurate enough, the uncertainty system is discussed in this example as follows

$$\frac{\partial w(x,t)}{\partial t}=\frac{\partial^2 w(x,t)}{\partial x^2}+\sum_{i=1}^{2}\mu_i(w(x,t))\big(A_i(x)+A_{ui}(x,t)\big)w(x,t)+B(x)u(x,t)+\upsilon(x,t),\tag{73}$$

where $A_{u1}(x,t)=0.1\sin(xt)$ and $A_{u2}(x,t)=0.2\cos(xt)$.

In this example, pick up $\rho_0=0.02$, $h=0.1$, the set of finite modes of Markov chain is $\mathcal{M}=\{1,2\}$, and the transfer rate matrix is $\Gamma_p=\begin{bmatrix}-1.5 & 1.5\\ 3 & -3\end{bmatrix}$. The position intervals are divided into two parts $[0,\pi]=\cup_{l=1}^{2}[\frac{(l-1)\pi}{2},\frac{l\pi}{2}]$ $(l=1,2)$, $\hat{x}_1=\frac{\pi}{4}$, $\hat{x}_2=\frac{3\pi}{4}$. According to Theorem 1, one can obtain the following parameters:

$$K_1(\frac{\pi}{4})=-2.0762,\quad K_1(\frac{3\pi}{4})=-2.1713,\quad K_2(\frac{\pi}{4})=-9.3311,\quad K_2(\frac{3\pi}{4})=-10.4299,\quad \gamma=9.5476.$$

The left one of Fig. 4 shows the spatiotemporal evolution of the system (72) state without input control. According to the left one of Fig. 4, it can be seen that the state of the system (72) is unstable without input control. The right one of Fig. 4 shows the spatiotemporal evolution of the state of system (73) under the proposed event-triggered controller.

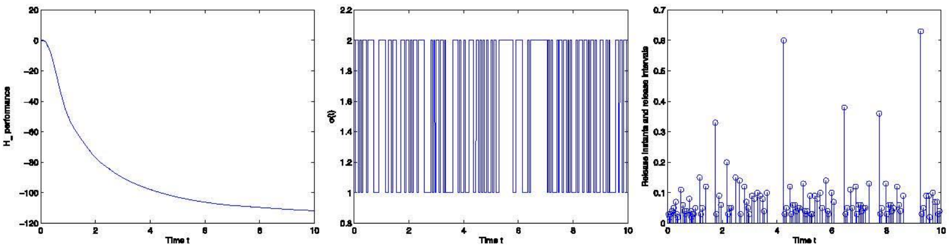


Fig. 5. The left figure shows the evolution of the variable $H(t) = \mathbf{E}\left\{\int_0^{10} ||w(x,t)||^2_{\mathcal{L}_2} dt\right\} - \gamma^2 \int_0^{10} ||\delta(x,t)||^2_{\mathcal{L}_2} dt$; the middle one shows the switching trajectory of the Markov jump mode; the right one shows the release instants and release intervals $int(t) = t_{k+1}h + \tau_{k+1} - t_k h - \tau_k$.

Furthermore, one can obtain that $H(t) = \mathbf{E}\left\{\int_0^{10} ||w(x,t)||^2_{\mathcal{L}_2} dt\right\} - \gamma^2 \int_0^{\infty} ||\delta(x,t)||^2_{\mathcal{L}_2} dt$. If $H(t) < 0$, which means that (20) holds, which indicates that the H_∞ control performance in (20) is ensured. It can be easily found from the spatiotemporal evolution that the system (73) under the controller of event triggered sampling data can quickly reach convergence. In addition, as shown in left one of Fig. 5, the evolution of the variable $H(t)$ is given, indicating that the system (73) satisfies the H_∞ performance. As shown in the middle one of Fig. 5, the stochastic response of Markov jump mode $\sigma(t)$ is given. According to event-triggered conditions (10), one can derive event-triggered instant $t_k h$ and the trigger time interval $t_{k+1}h + \tau_{k+1} - t_k h - \tau_k$, $k \in \mathbb{N}$. Compared with the traditional periodic sampling control, the control method based on event-triggered control scheme can significantly reduce the unnecessary sampling data transmission and controller update frequency. As shown in the right one of Fig. 5, the event triggered instant is shown on the horizontal axis, and the ordinate shows the trigger time interval.

6. Conclusion

This paper studies the adaptive event-based H_∞ control problem for a class of T-S fuzzy distributed parameter system with parameter uncertain and actuator fault. An event-triggered control scheme with adaptive threshold is proposed, which is superior to the event-triggered communication scheme with constant threshold in performance and can minimize the number of sampled data transmission. Furthermore, a T-S fuzzy controller based on the adaptive event-triggered sampled data is designed. Based on a new Lyapunov functional and inequality technique, the stochastic exponential stability criterion of the closed-loop system is obtained, and the controller parameters are designed. The new developed inequality can reduce the conservatism of the stability criterion. Finally, the effectiveness of the theoretical calculation results is verified by two numerical simulations, and the results are compared with the relevant literature in the simulation, showing that the methods and results in this paper are less conservative.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Trans. Syst. Man Cybern. 1 (1985) 116–132.
[2] K. Tanaka, T. Ikeda, H.O. Wang, Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs, IEEE Trans. Fuzzy Syst. 6 (2) (1998) 250–265.
[3] Y. Wu, R. Lu, P. Shi, et al., Sampled-data synchronization of complex networks with partial couplings and T-S fuzzy nodes, IEEE Trans. Fuzzy Syst. 26 (2) (2017) 782–793.
[4] X. Song, S. Xu, H. Shen, Robust H_∞ control for uncertain fuzzy systems with distributed delays via output feedback controllers, Inf. Sci. 178 (22) (2008) 4341–4356.
[5] S.C. Liu, S.F. Lin, LMI-based robust sliding control for mismatched uncertain nonlinear systems using fuzzy models, Int. J. Robust Nonlinear Control 22 (16) (2012) 1827–1836.

[6] H.N. Wu, J.W. Wang, H.X. Li, Fuzzy boundary control design for a class of nonlinear parabolic distributed parameter systems, IEEE Trans. Fuzzy Syst. 22 (3) (2013) 642–652.
[7] J.W. Wang, H.N. Wu, Exponential pointwise stabilization of semilinear parabolic distributed parameter systems via the Takagi–Sugeno fuzzy PDE model, IEEE Trans. Fuzzy Syst. 26 (1) (2016) 155–173.
[8] H. Özbay, A. Tannenbaum, A skew Toeplitz approach to the H_∞ optimal control of multivariable distributed systems, SIAM J. Control Optim. 28 (3) (1990) 653–670.
[9] B. Van Keulen, A state-space approach to H_∞-control problems for infinite-dimensional systems, in: Analysis and Optimization of Systems: State and Frequency Domain Approaches for Infinite-Dimensional Systems, Springer, Berlin, Heidelberg, 1993, pp. 46–71.
[10] P. Apkarian, D. Noll, Structured H_∞-control of infinite-dimensional systems, Int. J. Robust Nonlinear Control 28 (2018) 3212–3238.
[11] M. Garcia-Sanz, A. Huarte, A. Asenjo, A quantitative robust control approach for distributed parameter systems, Int. J. Robust Nonlinear Control 17 (2–3) (2007) 135–153.
[12] E. Fridman, Sampled-data distributed H_∞ control of transport reaction systems, SIAM J. Control Optim. 51 (2) (2013) 1500–1527.
[13] N.B. Am, E. Fridman, Network-based H_∞ filtering of parabolic systems, Automatica 50 (12) (2014) 3139–3146.
[14] H.N. Wu, Z.P. Wang, H.X. Li, Improved H_∞ sampled-data control for semilinear parabolic PDE systems, Int. J. Robust Nonlinear Control 29 (6) (2019) 1872–1892.
[15] W.H. Chen, S. Luo, W.X. Zheng, Sampled-data distributed H_∞ control of a class of 1-D parabolic systems under spatially point measurements, J. Franklin Inst. 354 (1) (2017) 197–214.
[16] Z. Wu, Y. Wu, Z.G. Wu, et al., Event-based synchronization of heterogeneous complex networks subject to transmission delays, IEEE Trans. Syst. Man Cybern. Syst. 48 (12) (2017) 2126–2134.
[17] Y. Wu, R. Lu, Event-based control for network systems via integral quadratic constraints, IEEE Trans. Circuits Syst. I, Regul. Pap. 65 (4) (2017) 1386–1394.
[18] K.J. Åström, B. Bernhardsson, Comparison of Periodic and Event Based Sampling for First-Order Stochastic Systems, in: Proceedings of the 14th IFAC World Congress, vol. 11, Citeseer, 1999, pp. 301–306.
[19] K.E. Arzén, A simple event-based PID controller, in: Proc. 14th IFAC World Congress 18 (2) (1999) 423–428.
[20] X. Song, M. Wang, S. Song, et al., Event-triggered reliable H_∞ control for a class of nonlinear distributed parameter systems within a finite-time interval, J. Franklin Inst. 357 (16) (2020) 11368–11393.
[21] J. Qiu, S.X. Ding, H. Gao, S. Yin, Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems, IEEE Trans. Fuzzy Syst. 24 (2) (2016) 388–400.
[22] H. Zhang, J. Liu, S. Xu, H_∞ load frequency control of networked power systems via an event-triggered scheme, IEEE Trans. Ind. Electron. 67 (8) (2020) 7104–7113.
[23] H. Zhang, J. Liu, S. Xu, Z. Zhang, Practical stabilization of networked Takagi-Sugeno fuzzy systems via improved Jense inequalities, IEEE Trans. Cybern. (2020) 1–10, https://doi.org/10.1109/TCYB.2020.3026375.
[24] L. Xie, Output feedback H_∞ control of systems with parameter uncertainty, Int. J. Control 63 (4) (1996) 741–750.
[25] R.F. Curtain, H. Zwart, An Introduction to Infinite Dimensional Linear Systems Theory, Springer, New York, 1995.
[26] A. Halanay, Differential Equations: Stability, Oscillations, Time Lags, Academic Press, New York, 1966.
[27] K. Liu, V. Suplin, E. Fridman, Stability of linear systems with general sawtooth delay, IMA J. Math. Control Inf. 27 (4) (2010) 419–436.
[28] A. Selivanov, E. Fridman, Distributed event-triggered control of diffusion semilinear PDEs, Automatica 68 (2016) 344–351.
[29] E. Fridman, A. Blighovsky, Robust sampled-data control of a class of semilinear parabolic systems, Automatica 48 (5) (2012) 826–836.
[30] X. Wang, M.D. Lemmon, Self-triggering under state-independent disturbances, IEEE Trans. Autom. Control 55 (6) (2010) 1494–1500.
[31] D. Yue, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2) (2013) 475–481.
[32] L. Zou, Z. Wang, D. Zhou, Moving horizon estimation with non-uniform sampling under component-based dynamic event-triggered transmission, Automatica 120 (2020) 109154.
[33] Q. Li, B. Shen, Z. Wang, T. Huang, J. Luo, Synchronization control for a class of discrete time-delay complex dynamical networks: a dynamic event-triggered approach, IEEE Trans. Cybern. 49 (5) (2019) 1979–1986.
[34] W.H. Heemels, M.C.F. Donkers, A.R. Teel, Periodic event-triggered control for linear systems, IEEE Trans. Autom. Control 58 (4) (2013) 847–861.
[35] C. Peng, J. Zhang, H. Yan, Adaptive event-triggering H_∞ load frequency control for network-based power systems, IEEE Trans. Ind. Electron. 65 (2) (2018) 1685–1694.
[36] H. Wu, J. Wang, H. Li, Exponential stabilization for a class of nonlinear parabolic PDE systems via fuzzy control approach, IEEE Trans. Fuzzy Syst. 20 (2) (2012) 318–329.

编　　号：20225046

中国科学院成都科技查新咨询中心

检　索　报　告　书

项目名称：　论文的收录和刊物的分区及影响因子情况

委 托 者：　南京审计大学　李慧慧

检索单位：　中国科学院成都科技查新咨询中心

委托日期：　2022 年 12 月 28 日

完成日期：　2022 年 12 月 28 日

第 1页

一、项目要点

应南京审计大学季慧慧的委托，检索其提交的 1 篇论文的收录和刊物的分区及影响因子情况。

二、检索情况

1. SCI-E 收录

检索系统:	Science Citation Index Expanded (SCI-EXPANDED)
检索年限:	——
检索策略:	标 题 =Event-generator-based H infinity control of fuzzy distributed parameter systems

2.影响因子

检索系统:	Journal Citation Reports（JCR）
检索年限:	2021
检索策略:	期刊名称=FUZZY SETS AND SYSTEMS

3.分区

检索系统:	中国科学院文献情报中心期刊分区表
检索年限:	2021
检索策略:	期刊名称=FUZZY SETS AND SYSTEMS

三、检索结果

（一）经检索科学引文索引数据库（Science Citation Index Expanded (SCI-EXPANDED)），提交的 1 篇论文被 SCI-E 数据库收录，记录见附件 1。

（二）经检索《Journal Citation Reports》和《中国科学院文献情报中心期刊分区表》数据库，刊物的影响因子及分区情况见附件 1。

检 索 人：任 波

检索单位：中国科学院成都科技查新咨询中心（盖章）

检索时间：2022 年 12 月 28 日

第 2页

附件 1

第 1 条，共 1 条

标题: Event-generator-based H infinity control of fuzzy distributed parameter systems

作者: Ji, HH (Ji, Huihui); Cui, BT (Cui, Baotong)

来源出版物: FUZZY SETS AND SYSTEMS 卷: 432 特刊: SI 页: 28-49 DOI: 10.1016/j.fss.2021.03.012

出版年: MAR 28 2022

Web of Science 核心合集中的 "被引频次": 1

被引频次合计: 1

入藏号: WOS:000759748600002

文献类型: Article

作者关键词: Distributed parameter systems; Adaptive event-triggered control; H-infinity control; T-S fuzzy model; Actuator fault; Stochastic exponential stability

KeyWords Plus: SAMPLED-DATA CONTROL; TRIGGERED CONTROL; STABILITY

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

[Cui, Baotong] Jiangnan Univ, Key Lab Adv Proc Control Light Ind, Minist Educ, Wuxi 214122, Jiangsu, Peoples R China.

[Cui, Baotong] Jiangnan Univ, Sch IoT Engn, Wuxi 214122, Jiangsu, Peoples R China.

通讯作者地址: Ji, HH (通讯作者)，Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

电子邮件地址: jihuihui2009@163.com; btcui@jiangnan.edu.cn

出版商: ELSEVIER

出版商地址: RADARWEG 29, 1043 NX AMSTERDAM, NETHERLANDS

Web of Science 类别: Computer Science, Theory & Methods; Mathematics, Applied; Statistics & Probability

研究方向: Computer Science; Mathematics

IDS 号: ZF7MF

ISSN: 0165-0114

eISSN: 1872-6801

影响因子：4.462

中国科学院文献情报中心期刊分区表：

期刊全称：	FUZZY SETS AND SYSTEMS		
期刊简称：	FUZZY SET SYST	ISSN：	0165-0114
年份：	2021年	综述：	否
	学科名称	分区	Top期刊
小类	COMPUTER SCIENCE, THEORY & METHODS计算机：理论方法	2	-
小类	MATHEMATICS, APPLIED应用数学	1	-
小类	STATISTICS & PROBABILITY统计学与概率论	2	-
大类	数学	1	是

科研项目 高教项目 科研成果 学术活动 高被引 校级项目申报 纵向项目申报 教研项目申报 入账办理 更多 季慧慧

成果列表

考核时间更新 新增 待确认网推论文

名称 模糊 审核状态 所属单位 第一作者姓名 模糊 更多 查询

	名称	所有作者	成果日期	成果类型	审核状态	操作
☐	Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems	季慧慧,徐胜元（外）,张贺（外）	2024-01-01	论文	学校通过	收录补录 转载补录
☐	Networked sampled-data control of distributed parameter systems via distributed sensor networks SCIE	季慧慧,Cui, Baotong（外）,Liu, Xinzhi（外）	2021-07-01	论文	学校通过	收录补录 转载补录
☐	Event-triggered predictor-based control of distributed parameter systems	季慧慧,崔宝同（外）,刘新芝（外）	2021-03-01	论文	学校通过	收录补录 转载补录
☐	2023年度江苏省自动化学会科学技术奖	张贺（外）,喻果（外）,黄书华（外）,季慧慧,彭岳（外）,蒲翠梅（外）	2024-02-23	获奖	学校通过	
☐	Finite-time quasi- synchronization of stochastic multilayer networks with energy consumption estimation	徐玉华,Wu Xiaoqun（外）,Wan Xiaoxiao（外）,季慧慧,周世超	2022-04-16	论文	学校通过	
☐	Event-generator-based H∞ control of fuzzy distributed parameter systems	季慧慧,崔宝同（外）	2022-03-28	论文	学校通过	收录补录 转载补录
☐	概率论与数理统计课程思政的探索与思考 知	季慧慧	2022-01-05	论文	学校通过	收录补录 转载补录

成果查看

基本信息 收录转载 依托项目 审核记录 修改记录

基本信息

名称	Event-generator-based H∞ control of fuzzy distributed parameter systems		
发表/出版时间	2022-03-28	发表/刊物论文集	Fuzzy sets and systems
教育部统计归属	科技类	期刊来源类型	外文期刊
论文类型	期刊论文	所属单位	数学学院（公共数学教学部）
刊物级别	SCI 一区		
SCI论文分区		SSCI分区	
影响因子		项目来源	其他课题
一级学科	数学	奖励级别	特别奖励期刊
字数	1 万字	刊型	特刊
是否符合审计提升	否	提升后级别	特别奖励期刊
文献类型			

详细信息

发表范围	国外学术期刊	学校署名	第一单位
卷期页		DOI	
ISSN号		CN号	
附件	FSS2021.pdf 下载 预览 检索02-季慧慧.pdf 下载 预览		

关闭

4. Event-triggered predictor-based control of distributed parameter systems (中科院 2 区、重点奖励一档) (1/3)

Received: 2 May 2020 | Revised: 5 September 2020 | Accepted: 28 September 2020 | IET Control Theory & Applications

DOI: 10.1049/cth2.12073

ORIGINAL RESEARCH PAPER

Event-triggered predictor-based control of distributed parameter systems

Huihui Ji[1] | Baotong Cui | XinZhi Liu[2]

[1] School of Statistics and Mathematics, Nanjing Audit University, 86 West Yushan Road, Nanjing 211815, China

[2] School of Applied Mathematics, University of Waterloo, Ontario N2L 3G1, Cananda

Correspondence
Huihui Ji, School of Statistics and Mathematics, Nanjing Audit University, 86 West Yushan Road, Nanjing 211815, China.
Email: jihuihui2009@163.com

Abstract
This paper deals with the point control problem for a class of distributed parameter systems with time varying delay induced by the network. To eliminate the effect of time delay, a predictor with the time-varying gain is designed to predict the state based on the sampled data. Meanwhile, the prediction error vanishes exponentially with the desired decay rate. To lighten greatly network loads and effectively improve the utilisation of the resource, an event-triggered communication scheme is proposed to determine the transmitting of necessary sampled data. Then, based on the point feedback controller, the exponential stability condition of the distributed parameter system with the event-triggered scheme is derived in the framework of linear matrix inequality. Furthermore, the feedback gain is given in this paper by using the Lyapunov–Krasovskii method where a novel Lyapunov–Krasovskii functional is constructed. The event-triggered time interval is presented to show the number of maximum allowable packet loss. Finally, an example of a food web model is given to illustrate the effectiveness of the obtained results.

1 | INTRODUCTION

A large body of industrial processes and physical ecological phenomena, such as heat transfer [1], electric and magnetic fields [2], particle diffusion [3] and food web model [4], are modelled as a class of system called distributed parameter systems (DPSs). The dynamic analysis and control of DPSs are more difficult than the lump parameter systems (LPSs) due to the spatial and temporal property, while it is more reasonable to use DPSs to depict the processing of a practical case than LPSs [5, 6]. That is why extensive efforts have been devoted to DPSs in these recent decades, and a great number of results and methods have been developed for DPSs, see the references [7–13] and the cited work therein.

In view of the wide use of teleoperated and networked systems, the control problem of systems with delays is ubiquitous and challenging. The time delays can be generated as an intrinsic part of the controlled physical process or by the implementation of a feedback loop. The predictor-based control method is an effective way to deal with the time delays, which has been investigated and applied successfully for several decades, especially for finite-dimensional systems, see [14, 15] and references therein. It is an interesting area, while it is just opening up for research on infinite-dimensional DPSs. Recently, as the first attempt for the reaction-diffusion network system with constant long input delay, a Smith predictor-based feedback design was proposed in [16] to compensate the time delay, which is based on the Smith predictor proposed in [17]. Since then, the predictor feedback control method was shown in [18] for DPSs with input delay described by transport partial differential equation (PDE). Very recently, the stabilisation problem for a class of linear DPSs with a known constant delay was discussed in [19], where a predictor-based point controller was constructed to solve this problem. To be realistic, time delay [20] in the practical systems should be time-varying delay other than constant delay due to the change of environment and the aging of components. The current study, which is inspired from the work in [19], aims to design a predictor to effectively compensate the time-varying delay induced by the network.

For the networked system, the capacity of the signal data in the channel is limited, and consequently how to reduce the network workload and save the node energy is a matter of great interest. To this end, an event-triggered communication scheme (ETCS) is proposed to determine whether the sampling data is necessarily to be transmitted. However, for the infinite dimensional case, very few works on event-triggered control of DPSs are available. Until recently, event-triggered control problem of DPSs was solved in [21] by using model reduction approach. The event-triggered-based control method can greatly lighten network loads and avoid wasting resources, however, for the systems with spatially dependent diffusion coefficients, it is not applicable directly. To compensate for this, a distributed event-triggered networked control approach was presented in [22] for a class of DPSs governed by semilinear diffusion PDE. Moreover, an event-driven scheme was used in [23] to reduce the frequency of the signal transmissions between the observer and the controller. It should be pointed out that the above studies [22, 23] fail to take into account the effect of network latency on the system, and the corresponding results cannot be applied to the control of distributed parameter systems with network time delay. Although the network time delay were involved in the work [19], it is unrealistic to consider that the network is a constant time delay. Worse still, the study in [19] is failure to consider that overmuch sampled-data transmission can cause network channel congestion and waste of resources.

Motivated by the above discussion, this paper focuses on the predictor-based point control problem for a class of DPSs governed by reaction-diffusion networks with time-varying delay induced by the network. To eliminate the effect of time-varying delay, a predictor with the time-varying gain is designed to predict the state. Meanwhile, the prediction error vanishes exponentially with the desired decay rate. Then, by using the point feedback controller based on the predictor, the exponential stability condition of the DPSs with the event-triggered communication scheme is derived in the framework of linear matrix inequality (LMI). Moreover, the feedback gain is given in this paper by using the Lyapunov–Krasovskii method where a novel Lyapunov–Krasovskii functional is constructed. The event-triggered time interval is presented to show the number of maximum allowable packet loss. Finally, a food web model example is shown to illustrate the effectiveness of the obtained results. The main contributions of this paper are summarised as follows: (1) we construct an improved predictor with the time-varying gain to deal with the time-varying delay induced by the network, which is different from the predictor in [19]; (2) a novel Lyapunov–Krasovskii functional candidate is constructed to derive the conditions guaranteeing exponential stability of prediction error systems; (3) an extended Jensen's inequality and some useful lemmas are developed to overcome the difficulties in the stability analysis process, which contribute to reduce the conservatism of stability conditions; (4) compared with the observer-based point control method used in [19], we tend to take advantage of ETCS for the networked system, and propose an ETCS to determine the transmitting of necessary sampled data, which can greatly lighten network loads and effectively improve the utilisation of the resource.

Notations: Throughout this paper, $\Omega = [0,\ l]$, $\Delta w(x,t) = \frac{\partial w(x,t)^2}{\partial x^2}$, $\nabla w(x,t) = \frac{\partial w(x,t)}{\partial x}$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathcal{L}^2(0,l)$ is the Hilbert space of square integrable functions over $(0,l)$ with values in $\mathbb{R}$ and one writes $\| w(x,t) \|_{\mathcal{L}^2} = \sqrt{\int_a^b w^T(x,t)w(x,t)dx} < \infty$. $\mathcal{H}^1(0,\ l) = w(x,t) \in \mathcal{L}^2(0,l), \nabla w^T(x,t) \in \mathcal{L}^2(0,l)$ is the Sobolev space and its norm by $\| w(x,t) \|_{\mathcal{H}^1} = \sqrt{\int_a^b w^T(x,t)w(x,t)dx + \int_a^b \nabla w^T(x,t)\nabla w(x,t)dx}$. The notation $X \geq Y$ (or $X > Y$) means that $X - Y$ is positive semi-definite (or positive definite) and the superscript $''T''$ stands the transpose. $\lambda_{\min}(X)$ is the minimum of all eigenvalues of matrix X, $\lambda_{\max}(\cdot)$ is the maximum of all eigenvalues. $\min\{a,b\}$ is the minimum of two real values a, b. $\max\{a,b\}$ is the maximum of two real values a, b. $\mathbb{N}_+$ denotes the set of positive integers. $\mathbb{N}$ means the set of integers.

2 | PRELIMINARIES AND PROBLEM FORMULATION

2.1 | Description of systems and problems

Consider a class of DPSs which can be modelled as follows:

$$\frac{\partial w(x,t)}{\partial t} = D \triangle w(x,t) + Aw(x,t) + u(x,t) \tag{1}$$

where $w(x,t) = [w_1(x,t),\ w_2(x,t), \dots,\ w_n(x,t)]^T$; the spatial variable $x \in \Omega = [0,\ l]$, and $w_i(x,t)$ $(i = 1, 2, \dots, n)$ represents the i_{th} sub-system satisfying $w_i(x,t) : \Omega \times (0, +\infty) \to \mathbb{R}$; n is the number of sub-systems in the DPSs; $\triangle w(x,t) = [\frac{\partial w_1^2(x,t)}{\partial x^2}, \frac{\partial w_2^2(x,t)}{\partial x^2}, \dots, \frac{\partial w_n^2(x,t)}{\partial x^2}]^T$; $D = \mathrm{diag}\{d_i\}_{i=1}^n$, the constant d_i is the diffusion coefficient of the i_{th} sub-system and is hence assumed to be positive; A is a constant matrix; $u(x,t) \in \mathbb{R}^n$ is the control input.

The boundary condition and initial value condition associated with the system (1) are given in form

$$w(0,t) = 0, \quad w(l,t) = 0, \quad t \in [0, +\infty) \tag{2}$$

$$w(x,0) = w_0(x), \quad (x) \in \Omega \tag{3}$$

where $w_0(x) \in \mathcal{H}^1(0,\ l)$.

Problem description: In this work, we consider the feedback control problem for a class of distributed parameter systems described by system (1) with network induced time-varying delays. In the process of feedback control, the output signal of the system needs to pass through a network and the time delay induced by the network is $\tau(t)$. When time delay $\tau(t)$ is larger, a feedback controller that relies on the delayed signal is difficult to control the system (1) effectively. The purpose of this paper is to design a predictor to compensate the network delay $\tau(t)$, meanwhile, energy saving and workload reducing need to be take into consideration when the control is processing. Thus, a predictor based on a event-triggered communication scheme is proposed to predict the state of the distributed parameter system (1) and to reduce unnecessary data transmission throughout the networked control system.

2.2 | Predictor based on event-triggered sampled data

Let Ω be divided into N_s sub-domains $\Omega_i, i \in \mathcal{V}_s = \{1, 2, ..., N_s\}$. For each sub-domain Ω_i, the sensor is set at $x_i \in \Omega_i$, then Ω_i is divided into Ω_i^L and Ω_i^R by spatial point x_i. And the length of Ω_i^L and Ω_i^R are denoted by γ_{iL} and γ_{iR}, respectively. We assume that: N_s in-domain sensors are assumed to provide point measurements of the state, which are sampled in time; the measured outputs are sampled at a constant period h; the sampling instants are listed as $\{jh|j = 1, 2, ...\} = S_1$; the event-triggered instants are listed as $\{t_k h|k = 1, 2, ...\} = S_2 \subset S_1$.

The predictor based on the event-triggered sampled data in this work is given as follows:

$$\begin{aligned}\frac{\partial \hat{w}(x,t)}{\partial t} &= D\Delta\hat{w}(x,t) + A\hat{w}(x,t) + u(x,t)\\ &+ Le^{-\alpha_0(t-t_k h)} \sum_{i\in\mathcal{V}_s} \chi_i(x)[\hat{w}(x_i, t_k h) - w(x_i, t_k h)]\end{aligned} \quad (4)$$

where $\hat{w}(x,t)$ be the prediction state, and $\chi_i(x) = \begin{cases} 1, & x \in \Omega_i, \\ 0, & x \notin \Omega_i. \end{cases}$, $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, $Le^{-\alpha_0(t-t_k h)}$ is the time-varying injection gain which used to ensure the prediction error decays with the rate α_0, and L will be designed later. Moreover, $\hat{w}(x,t) = 0$ for $t \in [-\infty, t_0 h + \tau_0)$, which means the predictor is not working in $t \in [-\infty, t_0 h + \tau_0)$ due to the time delay.

The boundary condition and initial value condition associated with the system (4) are given in form

$$\hat{w}(0,t) = 0, \quad \hat{w}(l,t) = 0, \quad t \in [0, +\infty) \quad (5)$$

$$\hat{w}(x,t) = \hat{w}_0(x,t), \quad x \in \Omega, \quad t \in (-\infty, t_0 h + \tau_0) \quad (6)$$

where $x \in \Omega$, $t \in [t_k h + \tau_k, \ t_{k+1}h + \tau_{k+1})$, $\hat{w}_0(x,t) \in \mathcal{H}^1(0, l)$, $t_0 h$ is the first event-triggered instant of the system (4).

The ETCS is used to determine the error between the predicted data and the original system data, which aims to ensure effectively predict the original system. If the error is within the allowable range, the new error data will not be transmitted; otherwise, it will be sent to the predictor. More specifically, the latest transmitted error data is $\bar{w}(x, t_k h) = \hat{w}(x, t_k h) - w(x, t_k h)$, whether the new error information $\bar{w}(x, t_k h + \ell h)$ between the plant and the predictor needs to be sent to the predictor depends on the following condition:

$$\begin{aligned}&\sum_{i=1}^{N_s}(\gamma_{iL} + \gamma_{iR})\epsilon^T(x_i, t_k h + \ell h)Q\epsilon(x_i, t_k h + \ell h)\\ &\le \sigma \sum_{i=1}^{N_s}(\gamma_{iL} + \gamma_{iR})\bar{w}^T(x_i, t_k h + \ell h)Q\bar{w}(x_i, t_k h + \ell h)\end{aligned} \quad (7)$$

with $\epsilon(x_i, t_k h + \ell h) = \bar{w}(x_i, t_k h + \ell h) - \bar{w}(x_i, t_k h)$, where $\sigma \in (0, 1)$, Q is a symmetric positive matrix, $\ell \in \mathbb{N}_+$. If the current error data violate the event-triggered threshold condition (7), the current error information will be sent to the predictor side; otherwise, this current error information will be discarded directly. Then the next release instant $t_{k+1}h$ ($k \in \mathbb{N}$) can be determined by

$$\begin{aligned}t_{k+1}h = (t_k + 1)h + \min_{\ell} \Bigg\{ \ell h | &\sum_{i=1}^{N_s}(\gamma_{iL} + \gamma_{iR})\epsilon^T(x_i, t_k h + \ell h)Q\epsilon(x_i, t_k h + \ell h) >\\ &\sigma \sum_{i=1}^{N_s}(\gamma_{iL} + \gamma_{iR})\bar{w}^T(x_i, t_k h + \ell h)Q\bar{w}(x_i, t_k h + \ell h) \Bigg\}.\end{aligned} \quad (8)$$

Remark 1. The event condition (7) with $\bar{w}(x, t_k h) \neq 0$ can be rewritten as

$$\begin{aligned}&\| Q^{\frac{1}{2}}(\bar{w}(x, t_k h) - \bar{w}(x, t_k h + \ell h)) \|_{\mathcal{L}^2}\\ &\le \| \sigma^{\frac{1}{2}} Q^{\frac{1}{2}} \bar{w}(x, t_k h + \ell h) \|_{\mathcal{L}^2}\end{aligned} \quad (9)$$

which implies that the relative error between the current sampled signal and the last transmitted signal less than a threshold. As mentioned in [24], the event condition (7) can overcome some drawbacks of another event-triggered scheme with the form

$$\| (y(x, t_k h) - y(x, t_k h + \ell h)) \|_{\mathcal{L}^2} \le \sigma \quad (10)$$

for example, when the state of the plant is approaching zero, there may exist an instant $t_k h$ such that (7) is satisfied for the threshold σ, which means there is no events to be triggered, then there is no signal to be updated after the instant $t_k h$. However, the event condition (7) proposed in this paper can make sure that the signal can be updated even after the instant $t_k h$. That is the main idea why we propose the event-triggered scheme (7). Moreover, the threshold $\sigma \in (0, 1)$ can be viewed as an index of measuring the relative error between the current sampled data and the last transmitted data. When the threshold σ infinitely close to zero, the ℓ in (7) should take the minimum value ℓ, then the next release instant $t_{k+1}h$ should be $t_k h + h$, which implies that the event-triggered control scheme reduces to the traditional periodic time-triggered control scheme. Therefore, the event-triggered control method in this paper show its advantage over time-triggered control approach used in [19] and take the method in [19] as a special case.

2.3 | Point control scheme

Now, we divide Ω into N_c new sub-domains denoted by $\Omega_j, j \in \mathcal{V}_c = \{1, 2, ..., N_c\}$. For each sub-domain Ω_j, the actuator is set at $\bar{x}_j \in \Omega_j$, then Ω_j is divided into Ω_j^L and Ω_j^R. And the length of Ω_j^L and Ω_j^R denote as η_{jL} and η_{jR}, respectively. It should be pointed out that this division is independent of the division for setting sensors, that is to say, it is possible that the point set $\mathcal{V}_c$ is independent of $\mathcal{V}_s$, and sensors and actuators are not necessary to be set at the same locations.

In this work, we consider the point controller modelled by the Dirac delta function in the following form:

$$u(x,t) = \sum_{j\in\mathcal{V}_c} \delta(x-\bar{x}_j)u_j(\bar{x}_j,t) \tag{11}$$

where $\delta(x)$ is the Dirac delta function representing point actuation; $\bar{x}_j, j \in \mathcal{V}_c$ is the position of the j_{th} actuator for each subdomain $\bar{x}_j \in \Omega_j, j \in \mathcal{V}_c$.

The control signals are chosen as

$$u_j(\bar{x}_j,t) = -K_j\hat{w}(\bar{x}_j,t),\ j\in\mathcal{V}_c \tag{12}$$

where K_j is the control gain which need to be designed.

Define $d(t) = t - t_k h$ and $\tau \le d(t) \le \tau_M = \max\{t_{k+1}h + \tau_{k+1} - t_k h - \tau_k\}$. Then, system (1) yields

$$\begin{aligned}\frac{\partial w(x,t)}{\partial t} &= D\Delta w(x,t) + Aw(x,t)\\ &\quad - \sum_{j\in\mathcal{V}_s}\delta(x-\bar{x}_j)K_j\hat{w}(\bar{x}_j,t_kh)\end{aligned} \tag{13}$$

where $x\in\Omega, t\in[t_kh+\tau,\ t_{k+1}h+\tau)$.

The boundary condition and initial value condition associated with the system (13) are given in form

$$w(0,t) = 0,\quad w(l,t)=0,\quad t\in[0,+\infty) \tag{14}$$

$$w(x,t) = w_0(x,t),\quad x\in\Omega,\quad t\in[-d(t),0] \tag{15}$$

where $w_0(x,t)\in\mathcal{H}^1(0,l)$.

The predictor dynamic model (4) with point controller can be constructed as the following form:

$$\begin{aligned}\frac{\partial \hat{w}(x,t)}{\partial t} &= D\triangle\hat{w}(x,t) + A\hat{w}(x,t)\\ &\quad + Le^{-\alpha_0(t-t_kh)}\sum_{i\in\mathcal{V}_s}\chi_i(x)[\hat{w}(x_i,t_kh) - w(x_i,t_kh)]\\ &\quad - \sum_{j\in\mathcal{V}_s}\delta(x-\bar{x}_j)K_j\hat{w}(x,t)\end{aligned} \tag{16}$$

for $t\in[t_kh+\tau_k,\ t_{k+1}h+\tau_{k+1})$ with the boundary and initial conditions

$$\hat{w}(0,t)=0,\quad \hat{w}(l,t)=0,\quad t\in[0,+\infty) \tag{17}$$

$$\hat{w}(x,t) = \hat{w}_0(x,t),\quad t\in(-\infty,\ t_0h+\tau_0]. \tag{18}$$

The solutions of (16) in weak sense has been demonstrated in [19].

3 | MAIN RESULTS

In the following, the synchronisation analysis between original system (1) and predictor (4) will be given, which aims is to study the effectiveness of prediction system. Then, the point feedback controller will be designed for the system (1).

First of all, some useful Lemmas are given as follows:

Lemma 1 (Wirtinger Inequality, [25]). *For $x\in\mathcal{H}^1(a,b)$, if $h(a)=h(b)=0$*

$$\| h(x)\|^2_{\mathcal{L}^2} \le \frac{(b-a)^2}{\pi^2}\left\|\frac{dh(x)}{dx}\right\|^2_{\mathcal{L}^2}$$

holds; and if $h(a)=0$ or $h(b)=0$

$$\| h(x)\|^2_{\mathcal{L}^2} \le \frac{4(b-a)^2}{\pi^2}\left\|\frac{dh(x)}{dx}\right\|^2_{\mathcal{L}^2}$$

holds.

Lemma 2 (Halanay Inequality, [26]). *Let $0<\alpha_2<\alpha_1$, $h\ge 0$, $d>0$ and $V(t):\ [t_0-d,\ \infty)\to[0,\ \infty)$ be an absolutely continuous function such that*

$$\dot{V}(t)\le -\alpha_1V(t) + \alpha_2\sup_{-h\le\xi\le 0}V(t+\xi),\quad \forall t\ge t_0 \tag{19}$$

holds. Then the following statement:

$$V(t)\le e^{-\alpha_0(t-t_0)}\sup_{-h\le\xi\le0}V(t_0+\xi),\quad \forall t\ge t_0 \tag{20}$$

holds, where α_0 is the unique positive solution of

$$\alpha_0 = \alpha_1 - \alpha_2e^{\alpha_0h}. \tag{21}$$

Lemma 3. *For $f(x)\in\mathcal{H}^1(a,b)$, there exists a positive symmetric matrix Q, scalars $\kappa_1>1$ and $\kappa_2>a_2-a_1$ such that*

$$\int_{a_1}^{a_2} f(x)^TQf(x)dx \le \min\{C_{v1},\ C_{v2}\} \tag{22}$$

where

$$\begin{aligned}C_{v1} &= \kappa_1(a_2-a_1)f(a_i)^TQf(a_i)\\ &\quad + \frac{4\kappa_1(a_2-a_1)^2}{\pi^2(\kappa_1-1)}\int_{a_1}^{a_2}\frac{df^T(x)}{dx}Q\frac{df(x)}{dx}dx\\ C_{v2} &= \kappa_2 f(a_i)^TQf(a_i)\\ &\quad + \frac{4\kappa_2(a_2-a_1)^2}{\pi^2(\kappa_2-a_2+a_1)}\int_{a_1}^{a_2}\frac{df^T(x)}{dx}Q\frac{df(x)}{dx}dx.\end{aligned}$$

Proof is given in Appendix A.

Lemma 4. *(Extended Jensen's Inequality.) For any positive definite matrix $M \in \mathbb{R}^{n\times n}$, any non-zero scalar α, scalars $a < b$ and a function $\omega(\cdot) \in C([a,b],\mathbb{R}^n)$, the following inequality holds:*

$$\int_a^b e^{\alpha(s-b)}\omega^T(s)M\omega(s)ds \geq \frac{\alpha}{e^{\alpha(b-a)}-1}\int_a^b \omega^T(s)dsM\int_a^b \omega(s)ds + \sum_{i=1}^{\infty}\frac{e^{-\alpha b}}{\varrho_i}\varphi_i^T(y)M\varphi_i(y) \tag{23}$$

where $\varrho_i = \int_a^b \phi_i^2(s)ds > 0$, $\varphi_i(y) = \int_a^b \phi_i(s)y(s)ds$, *and* $y(s) = e^{\frac{\alpha s}{2}}\omega(s) - \frac{\alpha e^{-\frac{\alpha s}{2}}}{e^{\alpha(b-a)}-1}\int_a^b \omega(s)ds$

$$\begin{cases} \phi_{2m}(s) = \left(s-\frac{a+b}{2}\right)^{2m} + \sum_{i=0}^{m-1} a_{mi}\left(s-\frac{a+b}{2}\right)^{2i} \\ \phi_{2m+1}(s) = \left(s-\frac{a+b}{2}\right)^{2m+1} + \sum_{i=0}^{m-1} b_{mi}\left(s-\frac{a+b}{2}\right)^{2i+1} \end{cases}$$

with the assumption that, for $i = 0,1,2,\ldots,m-1$

$$\int_a^b \phi_{2m}(s)\phi_{2i}(s)ds = \int_a^b \phi_{2m+1}(s)\phi_{2i+1}(s)ds = 0.$$

Moreover

$$\int_a^b e^{\alpha(s-b)}\frac{d\omega^T(s)}{ds}M\frac{d\omega(s)}{ds}ds \geq \frac{\alpha}{e^{\alpha(b-a)}-1}[\omega(b)-\omega(a)]^T M[\omega(b)-\omega(a)] + \sum_{i=1}^{\infty}\frac{1}{\varrho_i}\varphi_i^T(y)M\varphi_i(y) \tag{24}$$

holds.

Proof is given in Appendix B.

Lemma 5. *For any positive definite matrix $M \in \mathbb{R}^{n\times n}$, any positive scalar $\alpha > 0$, scalars $a < b$ and a vector-valued function $\omega(\cdot) : [a,b] \to \mathbb{R}^n)$ such that the integrations below are well defined, then*

$$\int_a^b e^{\alpha(s-b)}\frac{d\omega^T(s)}{ds}M\frac{d\omega(s)}{ds}ds \geq \frac{\alpha}{e^{\alpha(b-a)}-1}\left(\xi_1^T M\xi_1 + \xi_2^T M\xi_2 + 2\xi_1^T Y\xi_2\right) \tag{25}$$

holds with any matrix Y, and $\xi_1 = \omega(b)-\omega(s)$, $\xi_2 = \omega(s)-\omega(a)$, and

$$\begin{bmatrix} M & Y^T \\ Y & M \end{bmatrix} \geq 0. \tag{26}$$

Proof is given in Appendix C.

3.1 | Synchronisation analysis between original system (1) and predictor (4)

The purpose of this sub-section is to study the effectiveness of prediction system. It is clear that stability of observation error system can guarantee effectiveness of prediction system. Then, we will analyse the stability of considered error system.

Denote $\bar{w}(x,t)$ be the prediction error between systems (1) and (4), then it can be derived that

$$\frac{\partial \bar{w}(x,t)}{\partial t} = D\triangle\bar{w}(x,t) + A\bar{w}(x,t) \tag{27}$$

for $t \in [0, t_0 b + \tau_0)$; and for $t \in [t_k b + \tau_k,\ t_{k+1}b + \tau_{k+1})$

$$\frac{\partial \bar{w}(x,t)}{\partial t} = D\triangle\bar{w}(x,t) + A\bar{w}(x,t) + Le^{-\alpha_0 d(t)}\sum_{i\in\mathcal{V}_s}\chi_i(x)\bar{w}(x_i, t-d(t)) \tag{28}$$

with the initial and boundary conditions

$$\bar{w}(0,t) = 0, \quad \bar{w}(l,t) = 0, \quad t \in [0,+\infty) \tag{29}$$

$$\bar{w}(x,t) = \bar{w}_0(x,t), \quad x \in \Omega, \quad t \in [-d(t), 0] \tag{30}$$

where $\bar{w}_0(x,t) \in \mathcal{H}^1(0,\ l)$.

Remark 2. Since that sampling variables exist in predictor system [19], the error system cannot be defined in the time interval $[t_k b + r, t_{k+1}b + r]$. Obviously, the predictor model shown in [19] is difficult to be analysed.

For convenience, we define a new variable

$$z(x,t) = e^{\alpha_0 t}\bar{w}(x,t),\ x \in \Omega_i,\ i \in \mathcal{V}_s$$
$$m(x, t-d(t)) = z(x_i, t-d(t)) - z(x, t-d(t)).$$

Since

$$\frac{\partial z(x,t)}{\partial t} = \alpha_0 e^{\alpha_0 t}\bar{w}(x,t) + e^{\alpha_0 t}\frac{\partial \bar{w}(x,t)}{\partial t}$$

and based on (28), one can obtain

$$\frac{\partial z(x,t)}{\partial t} = D\triangle z(x,t) + (A-\alpha_0 I)z(x,t) + L\sum_{i\in\mathcal{V}_s}\chi_i(x)[z(x,t-\tau(t)) + m(x,t-\tau(t))] \tag{31}$$

for $t \in [t_k h + \tau_k,\ t_{k+1} h + \tau_{k+1})$, $k \in \mathbb{N}_+$.

Define the following variables: $\mathbf{e}_i \in \mathbb{R}^{n \times 7n}$ with the i_{th} element being identity matrix $I \in \mathbb{R}^{n\times n}$ and others being zero matrices, for example, $\mathbf{e}_2 = [0\ I\ 0\ 0\ 0\ 0\ 0]$

$$\mathcal{P} = -(P_1 + P_3)D - D(P_1 + P_3) + \alpha_1(P_2 D + DP_2);$$

$$\begin{aligned}
Y = {} & \mathbf{e}_1^T [P_1 A + A^T P_1 + (\alpha_1 + 2\alpha_0)I + S_1 + \frac{\pi_1^T}{l^2}\mathcal{P}]\mathbf{e}_1 \\
& + \mathrm{sym}\{\mathbf{e}_1^T P_1 L(\mathbf{e}_2 + \mathbf{e}_3)\} - \alpha_2 \mathbf{e}_2^T P_1 \mathbf{e}_2 \\
& + \mathbf{e}_4^T [\tau_k R_1 + (\tau_M - \tau_k) R_2]\mathbf{e}_4 + e^{-\alpha_1 \tau_k}\mathbf{e}_5^T (S_2 - S_1)\mathbf{e}_5 \\
& - e^{-\alpha_1 \tau_M}\mathbf{e}_6^T S_2 \mathbf{e}_6 - \frac{\alpha_1}{e^{\alpha_1 \tau_k} - 1}(\mathbf{e}_1 - \mathbf{e}_5)^T R_1 (\mathbf{e}_1 - \mathbf{e}_5) \\
& - \frac{\alpha_1 e^{-\alpha_1 \tau_k}}{e^{\alpha_1(\tau_M - \tau_k)} - 1}[(\mathbf{e}_5 - \mathbf{e}_2)^T R_2 (\mathbf{e}_5 - \mathbf{e}_2) \\
& + (\mathbf{e}_2 - \mathbf{e}_6)^T R_2 (\mathbf{e}_2 - \mathbf{e}_6) + \mathrm{sym}\{(\mathbf{e}_5 - \mathbf{e}_2)^T Y(\mathbf{e}_2 - \mathbf{e}_6)\}] \\
& + \mathrm{sym}\{(P_3 \mathbf{e}_1 + P_2 \mathbf{e}_4 + P_2 \mathbf{e}_4)^T [(A + \alpha_0 I)\mathbf{e}_1 \\
& + L\mathbf{e}_2 + L\mathbf{e}_3 - \mathbf{e}_4]\} \\
& + \sigma \mathbf{e}_7^T Q \mathbf{e}_7 - (\mathbf{e}_7 - \mathbf{e}_2 - \mathbf{e}_3)^T Q(\mathbf{e}_7 - \mathbf{e}_2 - \mathbf{e}_3).
\end{aligned}$$

Theorem 1. *For given positive scalars τ_M, τ_k, α_0, α_1, α_2, $\sigma \in (0, 1)$, there exist positive matrices $P_1, S_1, S_2, R_1, R_2, P_3$ and any matrices P_2, Y such that*

$$\begin{bmatrix} R_2 & Y^T \\ Y & R_2 \end{bmatrix} \geq 0 \qquad \mathcal{P} \leq 0,$$

$$Y - \frac{\alpha_2 \pi^2}{4\gamma_{i_L}^2}\mathbf{e}_3^T (P_2 D + DP_2)\mathbf{e}_3 < 0, \tag{32}$$

$$Y - \frac{\alpha_2 \pi^2}{4\gamma_{i_R}^2}\mathbf{e}_3^T (P_2 D + DP_2)\mathbf{e}_3 < 0$$

hold, then the solution of prediction error system (28) satisfies

$$\| \tilde{w}(x,t) \|_{\mathcal{H}^1} \leq \bar{C} e^{-\frac{(\alpha_c + 2\alpha_0)t}{2}} \| w(x,0) \|_{\mathcal{H}^1} \tag{33}$$

for some $\bar{C} > 0$, which means that state of system (1) can be validly predicted by system (4) where the decay rate α_0 is unique determined by (21) and $2\alpha_0 < \alpha_c$ is some positive scalar.

Furthermore, under the event-triggered scheme (7), if (33) is satisfied, then there exist σ, α_0, α_c, τ_k, and τ_M such that

$$\ell h \leq \frac{2\ln\left(1 + \sigma^{\frac{1}{2}}\right)}{\alpha_c + 2\alpha_0} \tag{34}$$

and

$$(\ell + 1)h \leq \tau_M - \tau_k. \tag{35}$$

Moreover, the maximum number of allowing packet loss can be obtained

$$\ell_{max} = \frac{2\ln\left(1 + \sigma^{\frac{1}{2}}\right)}{h(\alpha_c - 2\alpha_0)}. \tag{36}$$

Proof. First of all, we define the following variables:

$$\zeta_1 = \left[z^T(x,t)\ z^T(x,t-\tau(t))\ m^T(x,t-\tau(t))\right]^T$$

$$\zeta_2 = \left[\frac{\partial z^T(x,t)}{\partial t}\ z(x,t-\tau_k)^T\ z(x,t-\tau_M)^T\right]^T$$

$$\zeta = \left[\zeta_1^T\ \zeta_2^T\ z^T(x_i, t_k h + \ell h)\right]^T.$$

We suggest to construct the following Lyapunov–Krasovskii functional

$$V(t) = \sum_{i=1}^{6} V_i(t) \tag{37}$$

where

$$\begin{aligned}
V_1(t) &= \int_\Omega \zeta^T \mathbf{e}_1^T P_1 \mathbf{e}_1 \zeta dx \\
V_2(t) &= \int_\Omega \nabla z^T(x,t)(P_2 D + DP_2)\nabla z(x,t) dx \\
V_3(t) &= \int_\Omega \int_{t-\tau_k}^{t} e^{\alpha_1(s-t)}\zeta^T \mathbf{e}_1^T S_1 \mathbf{e}_1 \zeta ds dx \\
V_4(t) &= \int_\Omega \int_{t-\tau_M}^{t-\tau_k} e^{\alpha_1(s-t)}\zeta^T \mathbf{e}_1^T S_2 \mathbf{e}_1 \zeta ds dx \\
V_5(t) &= \int_\Omega \int_{t-\tau_k}^{t} \int_\theta^t e^{\alpha_1(s-t)}\zeta^T \mathbf{e}_4^T R_1 \mathbf{e}_4 \zeta ds d\theta dx \\
V_6(t) &= \int_\Omega \int_{\tau_k}^{\tau_M} \int_{t-\theta}^{t} e^{\alpha_1(s-t)}\zeta^T \mathbf{e}_4^T R_2 \mathbf{e}_4 \zeta ds d\theta dx.
\end{aligned}$$

Taking the partial derivative of $V(t)$, one has

$$\begin{aligned}
\dot{V}_1(t) + \alpha_1 V_1(t) = {} & 2\int_\Omega z^T(x,t)P_1 D \triangle z(x,t) dx \\
& + 2\int_\Omega \zeta^T \mathbf{e}_1^T P_1 (A + \alpha_0 I + \frac{\alpha_1}{2} I)\mathbf{e}_1 \zeta dx \\
& + 2\int_\Omega \zeta^T \mathbf{e}_1^T P_1 L \mathbf{e}_2 \zeta dx + 2\int_\Omega \zeta^T \mathbf{e}_1^T P_1 L \mathbf{e}_3 \zeta dx
\end{aligned} \tag{38}$$

where the equation is obtained by using integrating by parts and taking into account the boundary conditions

$$2\int_\Omega z^T(x,t)P_1 D\triangle z(x,t)dx$$
$$= 2z^T(x,t)P_1 D\nabla z(x,t)|_{x=0}^{l}$$
$$-2\int_\Omega \nabla z^T(x,t)P_1 D\nabla z(x,t)dx$$
$$= -2\int_\Omega \nabla z^T(x,t)P_1 D\nabla z(x,t)dx. \quad (39)$$

By using integrating by parts and taking into account the boundary conditions, one has

$$\dot{V}_2(t) + \alpha_1 V_2(t)$$
$$= 2\int_\Omega \nabla z^T(x,t)(P_2 D + DP_2)\frac{\partial \nabla z(x,t)}{\partial t}dx$$
$$+ \alpha_1 \int_\Omega \nabla z^T(x,t)(P_2 D + DP_2)\nabla z(x,t)dx$$
$$= -2\int_\Omega \triangle z^T(x,t)(P_2 D + DP_2)\mathbf{e}_4\zeta dx$$
$$+ \alpha_1 \int_\Omega \nabla z^T(x,t)(P_2 D + DP_2)\nabla z(x,t)dx. \quad (40)$$

$$\dot{V}_3(t) + \alpha_1 V_3(t) = \int_\Omega \zeta^T \mathbf{e}_1^T S_1 \mathbf{e}_1 \zeta dx$$
$$- e^{-\alpha_1 \tau_k}\int_\Omega \zeta^T \mathbf{e}_5^T S_1 \mathbf{e}_5 \zeta dx. \quad (41)$$

$$\dot{V}_4(t) + \alpha_1 V_4(t) = e^{-\alpha_1 \tau_k}\int_\Omega \zeta^T \mathbf{e}_5^T S_2 \mathbf{e}_5 \zeta dx$$
$$- e^{-\alpha_1 \tau_M}\int_\Omega \zeta^T \mathbf{e}_6^T S_2 \mathbf{e}_6 \zeta dx. \quad (42)$$

By using the extended Jensen's inequality Lemma 4, one has

$$\dot{V}_5(t) + \alpha_1 V_5(t) \le \tau_k \int_\Omega \zeta^T \mathbf{e}_4^T R_1 \mathbf{e}_4 \zeta dx$$
$$- \frac{\alpha_1}{e^{\alpha_1 \tau_k} - 1}\int_\Omega \zeta^T(\mathbf{e}_1 - \mathbf{e}_6)^T R_1(\mathbf{e}_1 - \mathbf{e}_6)\zeta dx. \quad (43)$$

By using the extended Jensen's inequality Lemmas 4 and 5, one has

$$\dot{V}_6(t) + \alpha_1 V_6(t) \le (\tau_M - \tau_k)\int_\Omega \zeta^T \mathbf{e}_4^T R_2 \mathbf{e}_4 \zeta dx$$
$$- \frac{\alpha_1 e^{-\alpha_1 \tau_k}}{e^{\alpha_1(\tau_M - \tau_k)} - 1}\int_\Omega \big[\zeta^T(\mathbf{e}_5 - \mathbf{e}_2)^T R_2(\mathbf{e}_5 - \mathbf{e}_2)\zeta$$
$$+ \zeta^T(\mathbf{e}_2 - \mathbf{e}_6)^T R_2(\mathbf{e}_2 - \mathbf{e}_6)\zeta$$
$$+ 2\zeta^T(\mathbf{e}_5 - \mathbf{e}_2)^T Y(\mathbf{e}_2 - \mathbf{e}_6)\zeta\big]dx \quad (44)$$

where Y is an any matrix satisfying $\begin{bmatrix} R_2 & Y^T \\ Y & R_2 \end{bmatrix} \ge 0$.

To deal with the term with respect to $\mathbf{e}_5\zeta$ in (44), we employ the descriptor method. Namely, from Equation (31), there exist the symmetric matrices P_3 and P_2 such that

$$0 = 2\int_\Omega [P_3\mathbf{e}_1\zeta + P_2\mathbf{e}_4\zeta]^T[D\triangle z(x,t) + (A + \alpha_0 I)\mathbf{e}_1\zeta$$
$$+ L\mathbf{e}_2\zeta + L\mathbf{e}_3\zeta - \mathbf{e}_4\zeta]dx. \quad (45)$$

Similar to Equation (45), one has

$$0 = 2\int_\Omega [D\triangle z(x,t) + (A + \alpha_0 I)\mathbf{e}_1\zeta + L\mathbf{e}_2\zeta$$
$$+ L\mathbf{e}_3\zeta - \mathbf{e}_4\zeta]^T D^{-1}P_2 D\mathbf{e}_4 dx. \quad (46)$$

Integrating by parts and taking into account the boundary conditions, we obtain

$$2\int_\Omega z^T(x,t)P_3^T D\triangle z(x,t)dx$$
$$= -2\int_\Omega \nabla z^T(x,t)P_3^T D\nabla z(x,t)dx. \quad (47)$$

Since $z(0,t) = z(l,t) = 0$, based on condition (32) $\mathcal{P} < 0$ and combining Lemma 1, one has

$$\int_\Omega \nabla z(x,t)\mathcal{P}\nabla z(x,t)dx \le \frac{\pi^2}{l^2}\int_\Omega \zeta^T \mathbf{e}_1^T \mathcal{P}\mathbf{e}_1\zeta dx. \quad (48)$$

From (38) to (48) and and combining ECT condition (7), one has

$$\dot{V}(t) \le -\alpha_1 V(t) + \sum_{i\in\mathcal{V}_s}\int_{\Omega_i^L}\zeta^T\Phi\zeta dx + \sum_{i\in\mathcal{V}_s}\int_{\Omega_i^R}\zeta^T\Phi\zeta dx$$
$$+ \alpha_2\sum_{i\in\mathcal{V}_s}\int_{\Omega_i}\zeta^T\mathbf{e}_2^T P_1\mathbf{e}_2\zeta dx. \quad (49)$$

For each Ω_i $(i \in \mathcal{V}_s)$, one has $m(x_i, t - \tau(t)) = 0$ and $\nabla m(x, t - \tau(t)) = -\nabla z(x, t - \tau(t))$. For each $x \in \Omega_i$ by using Lemma 1 one has

$$-\int_{\Omega_i^L}\nabla z^T(x, t-\tau(t))(P_2 D + DP_2)\nabla z(x, t-\tau(t))dx$$
$$\le -\frac{\pi^2}{4\gamma_{i_L}^2}\int_{\Omega_i^L}\zeta^T\mathbf{e}_3^T(P_2 D + DP_2)\mathbf{e}_3\zeta dx \quad (50)$$
$$-\int_{\Omega_i^R}\nabla z^T(x, t-\tau(t))(P_2 D + DP_2)\nabla z(x, t-\tau(t))dx$$
$$\le -\frac{\pi^2}{4\gamma_{i_R}^2}\int_{\Omega_i^R}\zeta^T\mathbf{e}_3^T(P_2 D + DP_2)\mathbf{e}_3\zeta dx. \quad (51)$$

Combining with Equations (50) and (51), it is derived for any α_2 with $0 < \alpha_2 < \alpha_1$ that

$$
\begin{aligned}
&-\alpha_2 \sup_{\theta\in[-\tau_M,\,0]} V(t+\theta) \leq -\alpha_2 \sum_{i\in\mathcal{V}_s}\int_{\Omega_i} \zeta^T \mathbf{e}_2^T P_1 \mathbf{e}_2 \zeta\, dx \\
&-\alpha_2 \sum_{i\in\mathcal{V}_s}\int_{\Omega_i} \nabla z^T(x, t-\tau(t))(P_2 D + D P_2)\times \\
&\nabla z(x, t-\tau(t))dx \\
&\leq -\sum_{i\in\mathcal{V}_s} \frac{\alpha_2\pi^2}{4\gamma_{i_L}^2}\int_{\Omega_i^L} \zeta^T \mathbf{e}_3^T (P_2 D + D P_2)\mathbf{e}_3 \zeta\, dx \\
&-\sum_{i\in\mathcal{V}_s} \frac{\alpha_2\pi^2}{4\gamma_{i_R}^2}\int_{\Omega_i^R} \zeta^T \mathbf{e}_3^T (P_2 D + D P_2)\mathbf{e}_3 \zeta\, dx. \qquad (52)
\end{aligned}
$$

According to the above analysis and combining condition (32), one has

$$
\begin{aligned}
&\dot V(t) + \alpha_1 V(t) - \alpha_2 \sup_{\theta\in[-\tau_M,\,0]} V(t+\theta) \\
&\leq \sum_{i\in\mathcal{V}_s}\int_{\Omega_i^L} \zeta^T \Phi \zeta\, dx + \sum_{i\in\mathcal{V}_s}\int_{\Omega_i^R} \zeta^T \Phi \zeta\, dx \\
&-\sum_{i\in\mathcal{V}_s} \frac{\alpha_2\pi^2}{4\gamma_{i_L}^2}\int_{\Omega_i^L} \zeta^T \mathbf{e}_3^T (P_2 D + D P_2)\mathbf{e}_3 \zeta\, dx \\
&-\sum_{i\in\mathcal{V}_s} \frac{\alpha_2\pi^2}{4\gamma_{i_R}^2}\int_{\Omega_i^R} \zeta^T \mathbf{e}_3^T (P_2 D + D P_2)\mathbf{e}_3 \zeta\, dx. \qquad (53)
\end{aligned}
$$

Based on condition (32), one has

$$
\dot V(t) + \alpha_1 V(t) - \alpha_2 \sup_{\theta\in[-\tau_M,\,0]} V(t+\theta) < 0. \qquad (54)
$$

Further, by using Lemma 2, one has

$$
V(t) \leq e^{-\alpha_c(t-t_0 b-\tau_0)} \sup_{\theta\in[-\tau_M,0]} V(t_0 b + \tau_0 + \theta) \qquad (55)
$$

for $t \geq t_0 + \tau_0$ where α_c is the unique positive solution of $\alpha_c = \alpha_1 - \alpha_2 e^{\alpha_c \tau_M}$.

Moreover, $V(t) = V(0)$ for $t < 0$. And for $t \in [0, t_0 + \tau_0)$, it is derived from model (31) with $L = 0$ according to system (27). There must exist a large enough c such that $\frac{dV(t)}{dt} \leq cV(t)$ for $t \in [0, t_0 b + \tau_0)$, which means that $V(t) \leq e^{ct} V(0)$ for $t \in [0, t_0 b + \tau_0)$. Therefore, one has

$$
\begin{aligned}
\sup_{\theta\in[-\tau_M,0]} V(t_0 b + \tau_0 + \theta) &\leq e^{ct} V(0) \\
&\leq C_V \parallel z(x,0) \parallel_{\mathcal{H}^1}^2 \qquad (56)
\end{aligned}
$$

for some bounded constant $C_V \geq 0$. Then, (55) and (56) yield

$$
\begin{aligned}
\parallel z(x,t) \parallel_{\mathcal{H}^1}^2 &\leq \frac{1}{\lambda_{\max}(P_1)} V(t) \\
&\leq \frac{C_V}{\lambda_{\max}(P_1)} e^{-\alpha_c t} \parallel z(x,0) \parallel_{\mathcal{H}^1}^2 . \qquad (57)
\end{aligned}
$$

Since $z(x,t) = e^{\alpha_0 t} \bar w(x,t)$ with $2\alpha_0 < \alpha_c$, one has

$$
\parallel \bar w(x,t) \parallel_{\mathcal{H}^1}^2 \leq \frac{C_V}{\lambda_{\max}(P_1)} e^{-(\alpha_c+2\alpha_0)t} \parallel w(x,0) \parallel_{\mathcal{H}^1}^2 \qquad (58)
$$

for any $t \geq 0$ with $\bar C = \frac{C_V}{\lambda_{\max}(P_1)}$, which proves the result (33).

Furthermore, the event-triggered control scheme (9) implies

$$
\begin{aligned}
&\parallel Q^{\frac{1}{2}} (\bar w(x, t_k b) - \bar w(x, t_k b + \ell b)) \parallel_{\mathcal{L}^2} \\
&\leq \parallel \sigma^{\frac{1}{2}} Q^{\frac{1}{2}} \bar w^T(x, t_k b + \ell b) \parallel_{\mathcal{L}^2} . \qquad (59)
\end{aligned}
$$

Due to (33) and (59), one can derive that $\hat w(x, t_k b)$ and $\hat w(x, t_k b + \ell b)$ have the same sign, then, the discussion needs to be divided into the following two cases.

(1) If $0 \leq\parallel Q^{\frac{1}{2}} \bar w(x, t_k b + \ell b) \parallel_{\mathcal{L}^2} - \parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{L}^2} \leq\parallel Q^{\frac{1}{2}} \epsilon(x, t_k b + \ell b) \parallel_{\mathcal{L}^2}$, then we have

$$
\begin{aligned}
&0 \leq (\sigma^{\frac{1}{2}} - 1) \parallel Q^{\frac{1}{2}} \bar w(x, t_k b + \ell b) \parallel_{\mathcal{L}^2} \\
&+ \parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{L}^2} \\
&\leq (\sigma^{\frac{1}{2}} - 1) e^{-\frac{\alpha_c - 2\alpha_0}{2}\ell b} \parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{H}_1} \\
&+ \parallel Q^{\frac{1}{2}} \bar w^T(x, t_k b) \parallel_{\mathcal{H}_1}, \qquad (60)
\end{aligned}
$$

which implies that $1 + (\sigma^{\frac{1}{2}} - 1) e^{-\frac{\alpha_c + 2\alpha_0}{2}\ell b} \geq 0$. Obviously, it holds for any ℓb.

(2) If $0 \leq\parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{L}^2} - \parallel Q^{\frac{1}{2}} \bar w(x, t_k b + \ell b) \parallel_{\mathcal{L}^2} \leq\parallel Q^{\frac{1}{2}} \epsilon(x, t_k b + \ell b) \parallel_{\mathcal{L}^2}$, then we have

$$
\begin{aligned}
&0 \leq (\sigma^{\frac{1}{2}} + 1) \parallel Q^{\frac{1}{2}} \bar w(x, t_k b + \ell b) \parallel_{\mathcal{L}^2} \\
&- \parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{L}^2} \\
&\leq (\sigma^{\frac{1}{2}} + 1) e^{-\frac{\alpha_c + 2\alpha_0}{2}\ell b} \parallel Q^{\frac{1}{2}} \bar w(x, t_k b) \parallel_{\mathcal{H}_1} \\
&- \parallel Q^{\frac{1}{2}} \bar w^T(x, t_k b) \parallel_{\mathcal{H}_1} \qquad (61)
\end{aligned}
$$

which implies that $(\sigma^{\frac{1}{2}}+1)e^{-\frac{\alpha_c+2\alpha_0}{2}\ell h}-1\geq 0$. Therefore, we have

$$\ell h \geq \frac{2\ln\left(1+\sigma^{\frac{1}{2}}\right)}{\alpha_c+2\alpha_0}. \tag{62}$$

The proof is now completed. □

Remark 3. Based on Theorem 1, it is clear that τ_M can be obtained, but the sampling period h is not obtained. In view of ETCS (8), one has

$$h=\tau_M-\tau_k-\min_{\ell}\{\ell h|$$
$$\sum_{i=1}^{N_s}(\gamma_{iL}+\gamma_{iR})\epsilon^T(x_i,t_kh+\ell h)Q\epsilon(x_i,t_kh+\ell h)$$
$$>\sigma\sum_{i=1}^{N_s}(\gamma_{iL}+\gamma_{iR})\bar{w}^T(x_i,t_kh+\ell h)Q\bar{w}(x_i,t_kh+\ell h)\}. \tag{63}$$

Then, in order to obtain h we need to know maximum of ℓh. In the following, the approach is given to obtain the maximum of ℓh.

3.2 | Design feedback controller

In this work, the controller in each Ω_j is chosen as

$$u_j(x,t)=-K_j\hat{w}(\bar{x}_j,t_kh),\quad \forall j\in\mathcal{V}_c. \tag{64}$$

It is worth mentioning that the subset Ω_j which includes the jth controller is independent of the sub-domains Ω_i.

First of all, we define the following variables

$$\psi_{L_{11}}=\frac{4\mu_j\eta_{jL}}{\pi^2(\kappa_1-1)}(QK_j+K_jQ)-QD-DQ;$$
$$\psi_{R_{11}}=\frac{4(1-\mu_j)\eta_{jR}}{\pi^2(\kappa_1-1)}(QK_j+K_jQ)-QD-DQ;$$
$$\psi_{L_{22}}=QA+A^TQ+M_1+\alpha_sQ-\frac{\mu_j}{\kappa_1\eta_{jL}}(\mathcal{Y}_j+\mathcal{Y}_j^T);$$
$$\psi_{R_{22}}=QA+A^TQ+M_1+\alpha_sQ-\frac{(1-\mu_j)}{\kappa_1\eta_{jR}}(\mathcal{Y}_j+\mathcal{Y}_j^T);$$
$$\bar{\psi}_{L_{11}}=\frac{4\mu_j\eta_{jL}^2}{(\kappa_{jL}-\eta_{jL})\pi^2}(\mathcal{Y}_j+\mathcal{Y}_j^T)-QD-DQ;$$
$$\bar{\psi}_{R_{11}}=\frac{4(1-\mu_j)\eta_{jR}^2}{(\kappa_{jR}-\eta_{jR})\pi^2}(\mathcal{Y}_j+\mathcal{Y}_j^T)-QD-DQ;$$
$$\bar{\psi}_{L_{22}}=QA+A^TQ+M_1+\alpha_sQ-\frac{\mu_j}{\kappa_{jL}}(\mathcal{Y}_j+\mathcal{Y}_j^T);$$
$$\bar{\psi}_{R_{22}}=QA+A^TQ+M_1+\alpha_sQ-\frac{(1-\mu_j)}{\kappa_{jR}}(\mathcal{Y}_j+\mathcal{Y}_j^T).$$

In the following, we will give the design of the control gain K_j in the following result.

Theorem 2. *For a given positive scalar $0<\sigma<1$, if there exit a scalar $\kappa_1>1$ and positive definite matrix Q, M_1, and M_2 such that*

$$(\sigma-1)Q+M_2<0,\quad \psi_{L_{11}}<0,\quad \psi_{R_{11}}<0$$
$$\psi_{L_{22}}<0,\quad \psi_{R_{22}}<0 \tag{65}$$

holds or if there exist scalars $\kappa_{jL}>\eta_{jL}$, $\kappa_{jR}>\eta_{jR}$, and positive definite matrix Q, M_1, and M_2 such that

$$(\sigma-1)Q+M_2<0,\quad \bar{\psi}_{L_{11}}<0,\quad \bar{\psi}_{R_{11}}<0$$
$$\bar{\psi}_{L_{22}}<0,\quad \bar{\psi}_{R_{22}}<0 \tag{66}$$

holds where all $\mathcal{Y}_i$, $j\in\mathcal{V}_c$ are auxiliary variables, then the solution $\hat{w}(x,t)$ of system (16) satisfies

$$\|\hat{w}(x,t)\|_{\mathcal{L}^2}\leq \hat{C}e^{\frac{-\min\{\alpha_c-2\alpha_0,\alpha_s\}t}{2}}\|w(x,0)\|_{\mathcal{H}^1} \tag{67}$$

for $t\geq 0$. This implies that (13) is exponentially stable, namely, the predictor-based point controller (11) can exponentially stabilise system (1).

Moreover, the controller gains K_j, $j\in\mathcal{V}_c$ are derived as $K_j=Q^{-1}\mathcal{Y}_j$.

Furthermore, under the event-triggered scheme (7), if (67) is satisfied, then then there exist σ, α_0, α_c, α_s, τ_k, and τ_M such that

$$\ell h\leq \frac{2\ln\left(1+\sigma^{\frac{1}{2}}\right)}{\min\{\alpha_c-2\alpha_0,\alpha_s\}} \tag{68}$$

and

$$(\ell+1)h\leq\tau_M-\tau_k. \tag{69}$$

Moreover, the maximum number of allowing packet loss can be obtained

$$\ell_{max}=\frac{2\ln\left(1+\sigma^{\frac{1}{2}}\right)}{h\min\{\alpha_c-2\alpha_0,\alpha_s\}}. \tag{70}$$

Proof. Consider the following Lyapunov–Krasovskii functional

$$\hat{V}(t)=\int_\Omega \hat{w}^T(x,t)Q\hat{w}(x,t)dx$$

which is well-defined continuous for the weak solution. Calculating the derivative $\dot{\hat{V}}$ along system (16), one has

$$\dot{\hat{V}}(t) = -2\int_{\Omega} \hat{w}_x^T(x,t)QD\hat{w}_x(x,t)dx$$
$$+ 2\int_{\Omega} \hat{w}^T(x,t)QA\hat{w}(x,t)dx + 2\sum_{i=1}^{N}\int_{\Omega_i} \hat{w}^T(x,t)Q\mathcal{F}_i dx$$
$$- 2\sum_{j=1}^{N} \hat{w}^T(\bar{x}_j,t)QK_j\hat{w}(\bar{x}_j,t) \tag{71}$$

with $\mathcal{F}_i = Le^{-\alpha_0 d(t)}\bar{w}(x_i,t_k h)$.

By Young's inequality, there exist a positive definite matrix M_1 such that

$$2\sum_{i\in\mathcal{V}_s}\int_{\Omega_i} \hat{w}^T(x,t)Q\mathcal{F}_i dx \le \int_{\Omega} \hat{w}^T(x,t)M_1\hat{w}(x,t)dx$$
$$+ \sum_{i\in\mathcal{V}_s}\int_{\Omega_i} \mathcal{F}_i^T QM_1^{-1}Q\mathcal{F}_i dx. \tag{72}$$

Then, one has

$$\sum_{i\in\mathcal{V}_s}\int_{\Omega_i} \mathcal{F}_i^T QM_1^{-1}Q\mathcal{F}_i dx$$
$$= \sum_{i\in\mathcal{V}_s}\int_{\Omega_i} e^{2\alpha_0 d(t)}\bar{w}^T(x_i,t_k h)L^T QM_1^{-1}QL\bar{w}(x_i,t_k h)dx$$
$$\le 2\int_{\Omega} e^{2\alpha_0 d(t)}\lambda_{max}\{L^T QM^{-1}QL\}||\bar{w}(x_i,t_k h)||_{\mathcal{H}^1}^2 dx$$
$$\le 2\int_{\Omega} e^{2\alpha_0 t}\lambda_{max}\{L^T QM^{-1}QL\}\Big(||\bar{w}(x,t_k h)||_{\mathcal{H}^1}^2$$
$$+ ||m(x,t_k h)||_{\mathcal{H}^1}^2\Big)dx$$
$$\le C_1 e^{2\alpha_0 t}V(x,t_k h)$$
$$\le C_1 e^{2\alpha_0 t - \alpha_c(t - t_0 h - \tau_0)} \sup_{0\le\theta\le\tau_M} V(x,t_k h - \theta)$$
$$\le C_2 e^{(2\alpha_0 - \alpha_c)t} \parallel w(\cdot,0)\parallel_{\mathcal{H}^1}^2 \tag{73}$$

for $t \ge 0$ and some bounded $C_1 > 0$ and $C_2 = C_V C_1 e^{\alpha_c(t_0 h + \tau_0)}$, where the first inequality is from the Young's inequality and the third inequality is from the following:

$$\int_{\Omega_i} m(x,t_k h)^T m(x,t_k h)dx$$
$$\le \frac{4\gamma_{iL}^2}{\pi^2}\int_{\Omega_i^L} \nabla z(x,t_k h)^T\nabla z(x,t_k h)dx$$
$$+ \frac{4\gamma_{iR}^2}{\pi^2}\int_{\Omega_i^R} \nabla z(x,t_k h)^T\nabla z(x,t_k h)dx$$
$$\le \frac{4\gamma_i^2}{\pi^2}\int_{\Omega_i} \nabla z(x,t_k h)^T\nabla z(x,t_k h)dx. \tag{74}$$

From ETC condition (7), one has

$$(\sigma - 1)\bar{w}^T(x,t_k h + \ell h)Q\bar{w}(x,t_k h + \ell h)$$
$$+ 2\bar{w}^T(x,t_k h + \ell h)Q\bar{w}(x,t_k h) - \bar{w}^T(x,t_k h)Q\bar{w}(x,t_k h)$$
$$>0. \tag{75}$$

Similar to (72), there exist a positive definite matrix M_2 such that

$$2\bar{w}^T(x,t_k h + lh)Q\bar{w}(x,t_k h)$$
$$\le \bar{w}^T(x,t_k h + lh)M_2\bar{w}(x,t_k h + lh)$$
$$+ \bar{w}^T(x,t_k h)QM_2^{-1}Q\bar{w}(x,t_k h). \tag{76}$$

Similar to (73), one has

$$\bar{w}^T(x,t_k h)(QM_2^{-1}Q + Q)\bar{w}(x,t_k h)$$
$$\le C_3 e^{(2\alpha_0 - \alpha_c)t}||w(\cdot,0)||_{\mathcal{H}^1}^2 \tag{77}$$

for $t \ge 0$ and some bounded $C_3 > 0$.

There exists a positive scalar $\mu_j \in (0,1)$ $(j \in \mathcal{V}_c)$ such that the following inequality holds by using Lemma 3 with $\kappa_1 > 1$ and Young's inequality:

$$- 2\hat{w}^T(\bar{x}_j,t)QK_j\hat{w}(\bar{x}_j,t)$$
$$= -\frac{\mu_j}{\eta_{jL}}\int_{\Omega_{jL}} \hat{w}^T(\bar{x}_j,t)(QK_j + K_j^T Q)\hat{w}(\bar{x}_j,t)dx$$
$$- \frac{1-\mu_j}{\eta_{jR}}\int_{\Omega_{jR}} \hat{w}^T(\bar{x}_j,t)(QK_j + K_j^T Q)\hat{w}(\bar{x}_j,t)dx$$
$$\le -\frac{\mu_j}{\kappa_1\eta_{jL}}\int_{\Omega_{jL}} \hat{w}^T(x,t)(QK_j + K_j^T Q)\hat{w}(x,t)dx$$
$$+ \frac{4\mu_j\eta_{jL}}{\pi^2(\kappa_1 - 1)}\int_{\Omega_{jL}} \nabla\hat{w}^T(x,t)(QK_j + K_j^T Q)\nabla\hat{w}(x,t)dx$$
$$+ \frac{4(1-\mu_j)\eta_{jR}}{\pi^2(\kappa_1 - 1)}\int_{\Omega_{jR}} \nabla\hat{w}^T(x,t)(QK_j + K_j^T Q)\nabla\hat{w}(x,t)dx$$
$$- \frac{1-\mu_j}{4\eta_{jR}}\int_{\Omega_{jR}} \hat{w}^T(x,t)(QK_j + K_j^T Q)\hat{w}(x,t)dx \tag{78}$$

and by using Lemma 3, there exist scalars $\kappa_{jL} > \eta_{jL}$ and $\kappa_{jR} > \eta_{jR}$ such that

$$- 2\hat{w}^T(\bar{x}_j,t)QK_j\hat{w}(\bar{x}_j,t)$$

$$\leq -\frac{\mu_j}{\kappa_{jL}} \int_{\Omega_j^L} \hat{w}^T(x,t)(QK_j + K_j^T Q)\hat{w}(x,t)dx$$

$$+ \frac{4\mu_j \eta_{jL}^2}{\pi^2(\kappa_{jL} - \eta_{jL})} \int_{\Omega_j^L} \nabla \hat{w}^T(x,t)(QK_j + K_j^T Q)\nabla \hat{w}(x,t)dx$$

$$+ \frac{4(1-\mu_j)\eta_{jR}}{\pi^2(\kappa_{jR} - \eta_{jR})} \int_{\Omega_j^R} \nabla \hat{w}^T(x,t)(QK_j + K_j^T Q)\nabla \hat{w}(x,t)dx$$

$$- \frac{1-\mu_j}{\eta_{jR}} \int_{\Omega_j^R} \hat{w}^T(x,t)(QK_j + K_j^T Q)\hat{w}(x,t)dx. \quad (79)$$

Combining Equations (58)–(78), one has

$$\dot{\hat{V}}(t) + \alpha_s \hat{V}$$

$$\leq \sum_{j\in\mathcal{V}_c} \left(\int_{\Omega_j^L} \nabla \hat{w}^T(x,t)\psi_{11} \nabla \hat{w}(x,t)dx \right.$$

$$+ \int_{\Omega_j^R} \nabla \hat{w}^T(x,t)\psi_{22} \nabla \hat{w}(x,t)dx$$

$$\left. + \int_{\Omega_j^L} \hat{w}^T(x,t)\psi_{33}\hat{w}(x,t)dx + \int_{\Omega_j^L} \hat{w}^T(x,t)\psi_{44}\hat{w}(x,t)dx \right)$$

$$+ \bar{w}^T(x, t_k b + \ell b)[(\sigma - 1)Q + M_2]\bar{w}(x, t_k b + \ell b)$$

$$+ (C_2 + C_3)e^{(2\alpha_0 - \alpha_s)t} \parallel w(\cdot, 0) \parallel_{\mathcal{H}^1}^2$$

$$= \sum_{j\in\mathcal{V}_c} \left(\int_{\Omega_j^L} \xi^T \Psi_L \xi dx + \int_{\Omega_j^R} \xi^T \Psi_R \xi dx \right)$$

$$+ (C_2 + C_3)e^{(2\alpha_0 - \alpha_s)t} \parallel w(\cdot, 0) \parallel_{\mathcal{H}^1}^2 \quad (80)$$

where $\psi_{L_{ij}}$ and $\psi_{R_{ij}}$ are given in Equation (65) and

$$\xi = \left[\nabla \hat{\omega}^T(x,t) \ \hat{\omega}^T(x,t)\right]^T$$
$$\Psi_L = \mathrm{diag}\{\psi_{L_{11}}, \psi_{L_{22}}\}, \quad \Psi_R = \mathrm{diag}\{\psi_{R_{11}}, \psi_{R_{22}}\}.$$

Combining Equations (58)–(77) and (79), it is derived that

$$\dot{\hat{V}}(t) + \alpha_s \hat{V}$$

$$\leq \sum_{j\in\mathcal{V}_c} \int_{\Omega_j^L} \xi^T \mathrm{diag}\{\bar{\psi}_{L_{11}}, \bar{\psi}_{L_{22}}\}\xi dx$$

$$+ \sum_{j\in\mathcal{V}_c} \int_{\Omega_j^R} \xi^T \mathrm{diag}\{\bar{\psi}_{R_{11}}, \bar{\psi}_{R_{22}}\}\xi dx$$

$$+ (C_2 + C_3)e^{(2\alpha_0 - \alpha_s)t} \parallel w(\cdot, 0) \parallel_{\mathcal{H}^1}^2 . \quad (81)$$

By applying Schur's complement to conditions (65) and (66), one has that

$$\dot{\hat{V}}(t) \leq -\alpha_s \hat{V} + (C_2 + C_3)e^{(2\alpha_0 - \alpha_s)t} \parallel w(\cdot, 0) \parallel_{\mathcal{H}^1}^2 . \quad (82)$$

If $(\alpha_c - 2\alpha_0) \neq \alpha_s$, the comparison principle implies that (67) holds. If the conditions of Theorem 2 hold for $(\alpha_c - 2\alpha_0) = \alpha_s$, they remain true for slightly larger $\alpha_s' > \alpha_s$, implying that (67) holds for α_s. □

4 | NUMERICAL SIMULATION

In this section, a spatial model of cyclically competing populations is given to illustrate the effectiveness of our results. Consider individuals of three species A, B, and C, The interactions between each individual and its nearest neighbors by positions exchanging at a rate ε or by cyclic competition can be shown as the following chemical reactions:

$$AB \xrightarrow{1} AA, \quad BC \xrightarrow{1} BB, \quad CA \xrightarrow{1} CC \quad (83)$$

$$AB \xrightarrow{\gamma} A\Theta, \quad BC \xrightarrow{\gamma} B\Theta, \quad CA \xrightarrow{\gamma} C\Theta \quad (84)$$

$$A\Theta \xrightarrow{\mu} AB, \quad B\Theta \xrightarrow{\mu} BB, \quad C\Theta \xrightarrow{\mu} CA. \quad (85)$$

Reactions (83) shows cyclic prey-predator scenario that specie A preys on specie B, and specie B consumes specie C, and specie C feeds on specie A, and the predominant species immediate reproduction after consumption. For simplicity, we assume that the rate of this reaction is 1 and three species are symmetric. Reactions (84) is the solely consumption scenario, where empty site Θ implies no reproduction of the predator. These reactions occur at a rate γ. Reactions (85) means the reproduction of individuals of three species A, B, and C, which happens at a rate μ. Note that these reactions are important for ecological systems. In (83), an individual reproduces when having consumed a prey, due to thereby increased fitness. In contrast, in reactions (85) reproduction depends solely on the availability of empty space.

Based on the spatial model constructed by [27], where the diffusion constant $D = 2\varepsilon N^{-1}$ reflects the random local exchanges of individuals, and diffusivity D kept fixed when $N \to \infty$ by using a continuum limit of large systems. The spatial system without considering the noise terms can be modeled as the following DPS:

$$\frac{\partial w_i(x,t)}{\partial t} = d_i \triangle w_i(x,t) + w_i(x,t)[\mu(1-\rho) + w_{i+1}$$
$$- (1+\gamma)w_{i+2}(x,t)] \quad (86)$$

where $\vec{w}(x,t) = [a, \ b, \ c]^T$ stands the densities of the three distinct species, the total density is denoted by $\rho = a + b + c$, and N is the system size.

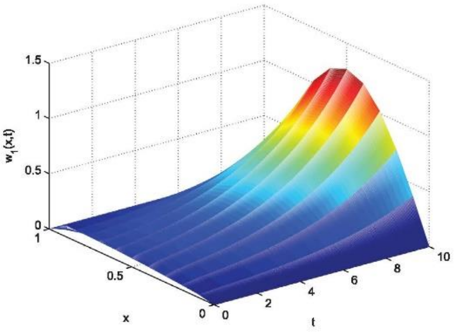


FIGURE 1 Profile of evolution of population A without controller

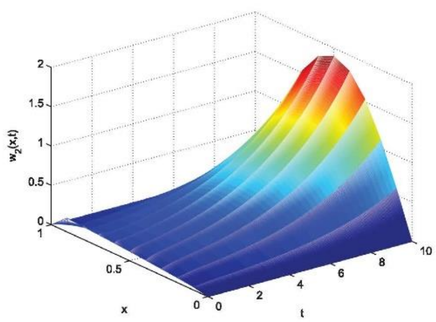


FIGURE 2 Profile of evolution of population B without controller

By applying variational method to system (86) at the point $w_i(x,t) = 0$, it is derived that

$$\frac{\partial w(x,t)}{\partial t} = D \triangle w(x,t) + Aw(x,t) \tag{87}$$

with $D = \mathrm{diag}\{d_1, d_2, d_3\}$ and $A = \mu(1-\rho)I$. It is obvious that if system (87) is stable, then system (86) is also stable.

Following (4), we consider the following dynamic of predictor:

$$\begin{aligned}\frac{\partial \hat{w}(x,t)}{\partial t} &= D \triangle \hat{w}(x,t) + A\hat{w}(x,t) \\ &\quad + Le^{\ell - t_k h} \sum_{i \in \mathcal{V}_s} (\hat{w}(x_i, t_k h) - w(x_i, t_k h))\end{aligned} \tag{88}$$

with $\mathcal{V}_s = \{1,2,3,4\}$.

Consider the parameters as follows:

$$D = 10^{-0.95}, \quad \gamma = \mu = 5.42, \quad \rho = 0.75.$$

Choose $\tau_M = 1.5$ and $\tau_k = 0.3$. Then by using Theorem 1, we choose $L = -1.04$. It is derived as $\ell h = 1.08$, i.e. $h = 0.12$ by using Theorem 1. Further, choose $u(x,t) = \sum_{j \in \mathcal{V}_c} K_j(\hat{w}(x_j,t) - w(x_j,t))$, $\mu = 0.5$, $\alpha_s = 1$, $\eta_{j_L} = \eta_{j_R} = 0.03$, $\kappa_1 = 2$, $\kappa_{j_L} = \kappa_{j_R} = 2\eta_{j_L}$. Then, one can obtain that $K_j = 0.3138$ for $j \in \mathcal{V}_c = \{1, \ldots, 10\}$ by using Theorem 2. And $h = 0.12$ can be derived by Theorem 2.

Simulation: Choose $w(x,0) = [0.1sin(\frac{\pi}{2}), 0.2sin(\frac{\pi}{3}), 0.3sin(\pi x)]^T$. The profiles of evolution for populations A, B, and C of cyclic competing populations are respectively shown in Figures 1, 2, and 3. Obviously, the system on cyclic competing populations is unstable. Under ETC, the profiles of evolution for prediction error systems of populations A, B, and C are respectively shown in Figures 4–6. It is clear that prediction error systems of populations A, B, and C are exponentially stable, which means that the predictor model is valid. Under the point controllers, the profiles of evolution for populations A, B, and C of cyclic competing populations are respectively shown in Figures 7–9. It is easy to see that the profiles exponentially converge to zero, that is to say, the point controllers are valid. Figure 10 shows release instants and release intervals; it is clear that the number of accepted packets by the event-based controller is less than

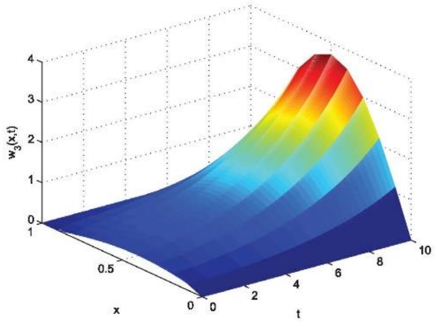


FIGURE 3 Profile of evolution of population C without controller

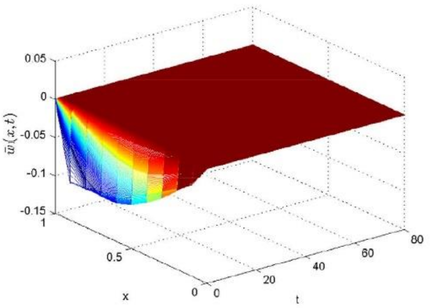


FIGURE 4 Profile of evolution of predicting error figure of population A

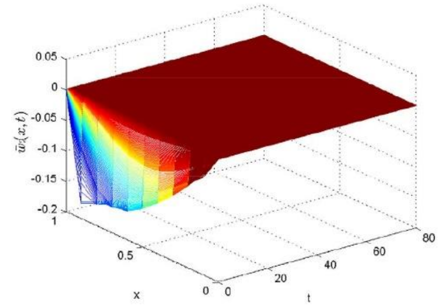


FIGURE 5 Profile of evolution of predicting error figure of population B

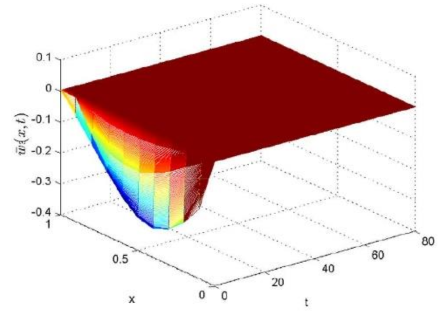


FIGURE 6 Profile of evolution of predicting error figure of population C

the one by using the periodic sampling control method, which means that our presented method is better.

Remark 4. It is easy to see from Figure 10 in this paper that the number of the sampled data accepted by the predictor is 58 with our presented method in the time interval [0,80] and the number is 666 using periodic sampled-data methods, which

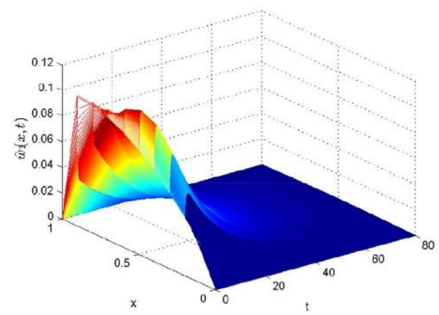


FIGURE 7 Profile of evolution of population A with controller

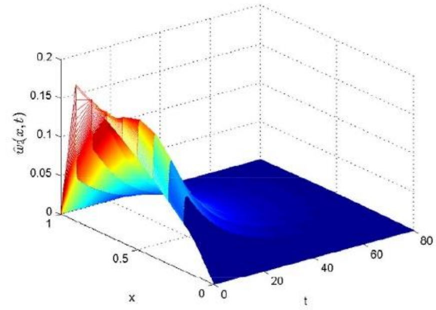


FIGURE 8 Profile of evolution of population B with controller

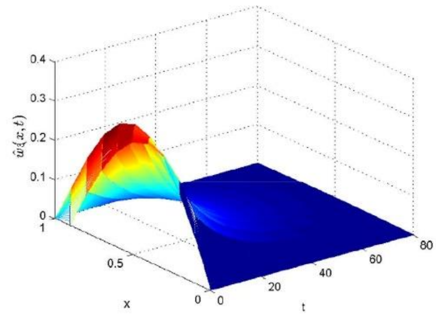


FIGURE 9 Profile of evolution of population C with controller

implies that our method is more economical of resources and superior. For clarity, the total amount and update frequency of the sampled data transmitted based on the event-triggered predictor, the fixed period predictor and the fixed period estimator respectively is compared by plotting the evolution

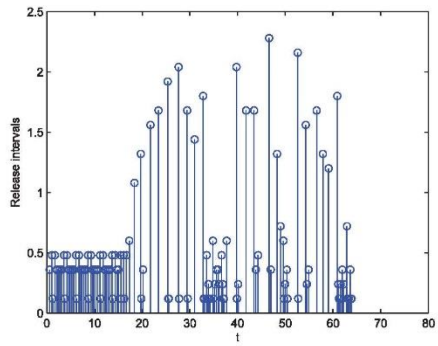


FIGURE 10 Release instants and release intervals

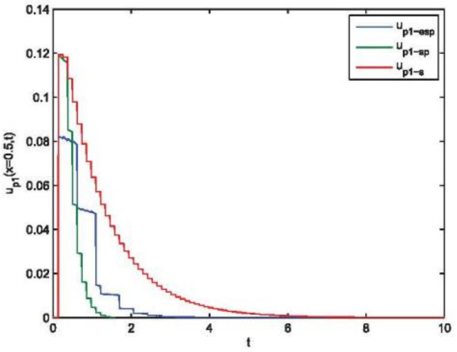


FIGURE 11 Profile of evolution of $u_{p1-eps} = Le^{-\alpha(t-t_k h)}w_1(0.5, t_k h)$, $u_{p1-sp} = Le^{-\alpha(t-kh)}w_1(0.5, kh)$ and $u_{p1-s} = Lw_1(0.5, kh)$

of the major components of the predictor/estimator in time interval [0,10]. Pick the space point $x = 0.5$, then the major components of the predictor/estimator by using our method, periodic predictor-based scheme and periodic estimator-based scheme without predictor are $u_{p1-eps} = Le^{-\alpha(t-t_k h)}w_1(0.5, t_k h)$, $u_{p1-ps} = Le^{-\alpha(t-kh)}w_1(0.5, kh)$ and $u_{p1-s} = Lw_1(0.5, kh)$, respectively, where h is the sampling periodic, kh are the sampling instants, $t_k h$ are the triggered instants.

In Figure 11, according to our method, the periodic predictor-based scheme and the periodic estimator-based scheme without predictor, the shortest convergence time of u_{p1} is 1.5 seconds, 3 seconds, and 7 seconds, respectively. Obviously, the shortest convergence time based on our method and the periodic predictor-based scheme quite close, which is significantly less than one based on the the periodic estimator-based scheme without predictor. Meanwhile, the number of updating sampled-data in Figure 11 using our method is the least than others, which implies that our method is superior.

5 | CONCLUSION

In this paper, a class of distributed parameter system with network time-varying delay is studied. In order to eliminate the time-varying delay and reduce the unnecessary transmission of sampled data, an event-triggered estimator was proposed. The estimated error converges exponentially with the expected decay rate to eliminate the effect of input delay in the system. By using Lyapunov method and LMI, the exponential stability criterion of the closed-loop system based on predictive control is derived. In addition, the feedback gain and the maximum number of packets allowed to be lost are given. Some improved inequalities are used to reduce the conservatism of the exponential stability criterion. Finally, a food web model is used to illustrate the validity of the results. When considering some practical application scenes of the distributed parameter systems including sensors/actuators failure and boundary control, the future work will study the problems of boundary control and fault-tolerant control [28, 29].

REFERENCES

1. Bejan, A.: Convection Heat Transfer. John Wiley & Sons, New Jersey (2013)
2. Binns, K.J., Lawrenson, P.J.: Analysis and Computation of Electric and Magnetic Field Problems: Pergamon International Library of Science, Technology, Engineering and Social Studies. Elsevier, Amsterdam (2013)
3. Han, L., et al.: Economic dispatch optimization algorithm based on particle diffusion. Energy Convers. Manage. 105, 1251–1260 (2015)
4. Wang, J.-L., Wu, H.-N.: Passivity of delayed reaction–diffusion networks with application to a food web model. Appl. Math. Comput. 219(24), 11311–11326 (2013)
5. Pérez-Pirela, M., et al.: Development of a simplified model for a distributed-parameter heat exchange system for thermodynamic principles-based control purposes. IFAC-PapersOnLine 51(13), 396–401 (2018)
6. Qiu, J., et al.: Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems. IEEE Trans. Fuzzy Syst. 24(2), 388–400 (2015)
7. Kang, W., Fridman, E.: Distributed sampled-data control of Kuramoto–Sivashinsky equation. Automatica 95, 514–524 (2018)
8. Ren, Y.-Q., et al.: Dynamic switching based fuzzy control strategy for a class of distributed parameter system. J. Process Control 24(3), 88–97 (2014)
9. L.H. X. e. a., et al.: A novel three-dimensional fuzzy modeling method for nonlinear distributed parameter systems. IEEE Trans. Fuzzy Syst. 27(3), 489–501 (2019)
10. Wu, J.-W., et al.: Design of distributed H_∞ fuzzy controllers with constraint for nonlinear hyperbolic PDE systems. Automatica 48(10), 2535–2543 (2012)
11. D. X. e. a., et al.: Iterative learning control for distributed parameter systems based on non-collocated sensors and actuators. IEEE/CAA J. Autom. Sin. (2019)
12. He, W., et al.: Dynamical modeling and boundary vibration control of a rigid-flexible wing system. IEEE/ASME Trans. Mechatron. 25(6), 2711–2721 (2020)
13. He, W., et al.: Reinforcement learning control of a flexible two-link manipulator: an experimental investigation. IEEE Trans. Syst. Man Cybern. Syst. (2020)
14. Ahmed-Ali, T., et al.: Global exponential sampled-data observers for nonlinear systems with delayed measurements. Syst. Control Lett. 62(7), 539–549 (2013)
15. Pyrkin, A.A., Bobtsov, A.A.: Adaptive controller for linear system with input delay and output disturbance. IEEE Trans. Autom. Control 61(12), 4229–4234 (2016)
16. Krstic, M.: Control of an unstable reaction– diffusion PDE with long input delay. Syst. Control Lett. 58(10-11), 773–782 (2009)
17. Smith, O.J., A controller to overcome dead time. ISA J. 6, 28–33 (1959)
18. Krstic, M., Bekiaris-Liberis, N.: Compensation of infinite-dimensional input dynamics. Annual Rev. Control 34(2), 233–244 (2010)
19. Selivanov, A., Fridman, E.: Delayed point control of a reaction–diffusion PDE under discrete-time point measurements. Automatica 96, 224–233 (2018)
20. Choi, J.-Y., Krstic, M.: Compensation of time-varying input delay for discrete-time nonlinear systems. Int. J. Robust Nonlinear Control 26(8), 1755–1776 (2016)
21. Yao, Z., El-Farra, N.H.: Resource-aware model predictive control of spatially distributed processes using event-triggered communication. In: 52nd IEEE Conference on Decision and Control. pp. 3726–3731. IEEE, Piscataway (2013)
22. Selivanov, A., Fridman, E.: Distributed event-triggered control of diffusion semilinear PDEs. Automatica 68, 344–351 (2016)

23. Jiang, Z., et al.: Event-driven observer-based control for distributed parameter systems using mobile sensor and actuator. Comput. Math. Appl. 72(12), 2854–2864 (2016)
24. Liu, Q., et al.: Event-based recursive distributed filtering over wireless sensor networks. IEEE Trans. Autom. Control 60(9), 2470–2475 (2015)
25. Hardy, G., Littlewood, J.: Inequalities. Cambridge University Press, Cambridge (1952)
26. Halanay, A., Halanay, A.: Differential Equations: Stability, Oscillations, Time Lags. Elsevier, (1966), vol. 6, no. 5
27. Reichenbach, T., Frey, E.: Instability of spatial patterns and its ambiguous impact on species diversity. Phys. Rev. Lett. 101(5), 058102 (2008)
28. Wang, X., Yang, G.-H.: Fault-tolerant consensus tracking control for linear multiagent systems under switching directed network. IEEE Trans. Cybern. 50(5), 1921–1930 (2019)
29. Wang, X., Yang, G.-H.: Cooperative adaptive fault-tolerant tracking control for a class of multi-agent systems with actuator failures and mismatched parameter uncertainties. IET Control Theory Appl. 9(8), 1274–1284 (2015)
30. Kim, J.-H.: Further improvement of Jensen inequality and application to stability of time-delayed systems. Automatica 64, 121–125 (2016)
31. Park, P., et al.: Reciprocally convex approach to stability of systems with time-varying delays. Automatica 47(1), 235–238 (2011)

How to cite this article: Ji H, Cui B, Liu X. Event-triggered predictor-based control of distributed parameter systems. *IET Control Theory Appl.* 2020;1–16. https://doi.org/10.1049/cth2.12073

APPENDIX A: PROOF OF LEMMA 3

According to Lemma 1, one has

$$\begin{aligned}&\int_{a_1}^{a_2}[f(x)-f(a_i)]^T Q[f(x)-f(a_i)]dx\\&\leq\frac{4(a_2-a_1)^2}{\pi^2}\int_{a_1}^{a_2}\frac{df^T(x)}{dx}Q\frac{df(x)}{dx}dx.\end{aligned}\tag{89}$$

Moreover, it is derived that

$$\begin{aligned}&\int_{a_1}^{a_2}f^T(x)Qf(x)+f^T(a_i)Qf(a_i)dx\\&\leq 2\int_{a_1}^{a_2}f^T(x)Qf(a_i)dx\\&\quad+\frac{4(a_2-a_1)^2}{\pi^2}\int_{a_1}^{a_2}\frac{df^T(x)}{dx}Q\frac{df(x)}{dx}dx.\end{aligned}\tag{90}$$

Since Q is symmetric, one has $Q=Q^{\frac{1}{2}}Q^{\frac{1}{2}}$. Then, by using Young's inequality there exist constants $\kappa_1>1$ and $\kappa_2>a_2-a_1$ such that

$$\begin{aligned}2\int_{a_1}^{a_2}f^T(x)Qf(a_i)dx&\leq\frac{1}{\kappa_1}\int_{a_1}^{a_2}f^T(x)Qf(x)dx\\&\quad+\kappa_1(a_2-a_1)f^T(a_i)Qf(a_i)\end{aligned}\tag{91}$$

and

$$\begin{aligned}&2\int_{a_1}^{a_2}f^T(x)Qf(a_i)dx\\&\leq\frac{1}{\kappa_2}\int_{a_1}^{a_2}f^T(x)dxQ\int_{a_1}^{a_2}f(x)dx+\kappa_2 f^T(a_i)Qf(a_i)\\&\leq\frac{a_2-a_1}{\kappa_2}\int_{a_1}^{a_2}f^T(x)Qf(x)dx+\kappa_2 f^T(a_i)Qf(a_i)\end{aligned}\tag{92}$$

hold.

Substituting Equation (91) into Equation (90), and substituting Equation (92) into Equation (90), the result (22) can be obtained.

This proof is completed. □

APPENDIX B: PROOF OF LEMMA 4

Construct an auxiliary function as follows:

$$\begin{aligned}z(u)&=\int_u^b e^{\frac{\alpha s}{2}}\omega(s)ds+\frac{e^{\frac{-\alpha b}{2}}-e^{\frac{-\alpha u}{2}}}{e^{-\alpha a}-e^{-\alpha b}}\int_a^b\omega(s)ds\\&\quad+c\left(e^{\frac{-\alpha b}{2}}-e^{\frac{-\alpha u}{2}}\right)\theta\end{aligned}\tag{93}$$

where c is a scalar and $\theta\in\mathbf{R}^n$ is a vector, and they will be determined later. Then, one has

$$\begin{aligned}z(b)&=0;\\z(a)&=\int_a^b e^{\frac{\alpha s}{2}}\omega(s)ds+\frac{1}{e^{\frac{-\alpha b}{2}}+e^{\frac{-\alpha a}{2}}}\int_a^b\omega(s)ds\\&\quad+c\left(e^{\frac{-\alpha b}{2}}-e^{\frac{-\alpha a}{2}}\right)\theta;\\\frac{\partial z(u)}{\partial u}&=-e^{\frac{\alpha u}{2}}\omega(u)+\frac{\frac{\alpha}{2}e^{\frac{-\alpha u}{2}}}{e^{-\alpha a}-e^{-\alpha b}}\int_a^b\omega(s)ds\\&\quad+\frac{\alpha c}{2}e^{\frac{-\alpha u}{2}}\theta.\end{aligned}\tag{94}$$

Based on lemma 1 in [30], it is easy to see that for $u\in[a,b]$

$$\begin{aligned}&\int_a^b\frac{\partial z(u)}{\partial u}^T M\frac{\partial z(u)}{\partial u}du\\&=\int_a^b e^{\alpha u}\omega(u)^T M\omega(u)du\\&\quad-\frac{3\alpha}{4(e^{-\alpha a}-e^{-\alpha b})}\left[\int_a^b\omega(u)du\right]^T M\left[\int_a^b\omega(u)du\right]\end{aligned}$$

$$
-\frac{c\alpha}{2}\left[\int_a^b \omega(u)du\right]^T M\theta + \frac{c^2\alpha(e^{-\alpha a} - e^{-\alpha b})}{4}\theta^T M\theta
$$

$$
\geq \sum_{i=1}^{\infty} \frac{1}{\varrho_i}\varphi_i^T(y)M\varphi_i(y). \tag{95}
$$

Choose $\theta = \int_a^b \omega(u)du$. Then, (95) yields to

$$
\int_a^b e^{\alpha u}\omega(u)^T M\omega(u)du
$$

$$
\geq \left[\frac{\alpha}{e^{-\alpha a} - e^{-\alpha b}} - \frac{\alpha(e^{-\alpha a} - e^{-\alpha b})}{4}\left(c - \frac{1}{e^{-\alpha a} - e^{-\alpha b}}\right)^2\right] \times
$$

$$
\left[\int_a^b \omega(u)du\right]^T M \left[\int_a^b \omega(u)du\right]
$$

$$
+\sum_{i=1}^{\infty} \frac{1}{\varrho_i}\varphi_i^T\left(\frac{\partial z(u)}{\partial u}\right)M\varphi_i\left(\frac{\partial z(u)}{\partial u}\right). \tag{96}
$$

It is clear that when $c = \frac{1}{e^{-\alpha a} - e^{-\alpha b}}$, (96) yields to (23). This proof is completed. □

APPENDIX C: PROOF OF LEMMA 5

Define $g(s) = \frac{\alpha}{e^{\alpha(s-a)} + e^{\alpha(b-s)} - 2}$ for $a \leq s \leq b$. It is easy to see that

$$
\frac{d}{ds}g(s) = -\frac{\alpha^2(e^{\alpha(s-a)} - e^{\alpha(b-s)})}{(e^{\alpha(s-a)} + e^{\alpha(b-s)} - 2)^2}, \tag{97}
$$

which means that function $g(s)$ is monotonically increasing for $a \leq s \leq \frac{a+b}{2}$, and it is monotonically decreasing for $\frac{a+b}{2} \leq s \leq b$. Namely, $g(a) \leq g(s) \leq g(\frac{a+b}{2})$.

Then, according to Lemma 4, one has

$$
\int_a^b e^{\alpha(s-b)}\frac{d\omega^T(s)}{ds}M\frac{d\omega(s)}{ds}ds
$$

$$
\geq g(s)\left[\frac{\alpha}{g(s)(e^{\alpha(b-s)} - 1)}\xi_1^T M\xi_1\right.
$$

$$
\left.+\frac{\alpha}{g(s)(e^{\alpha(s-a)} - 1)}\xi_2^T M\xi_2\right]
$$

$$
\geq g(a)\left(\xi_1^T M\xi_1 + \xi_2^T M\xi_2 + 2\xi_1^T Y\xi_2\right), \tag{98}
$$

where the last inequality is from the reciprocally convex approach [31] with (26).

This proof is completed. □

南京理工大学图书馆

检 索 报 告

委托人： 季慧慧

委托人单位： 南京审计大学

检索要求： 季慧慧以第一作者发表的论文:1、在 SCIE 数据库中被收录情况；2、论文发表期刊的分区情况

检索数据库： SCIE（科学引文索引扩展版）
中国科学院文献情报中心期刊分区表（2021）

检索时间： 2023 年 8 月 27 日

检 索 结 果

本次检索对季慧慧提供的论文在 SCIE 数据库中被收录情况及论文发表期刊在中国科学院文献情报中心期刊分区表上的分区情况进行了检索，结果如下：

1、收录情况

季慧慧以第一作者发表的论文被 SCIE 数据库收录 2 篇。

2、分区情况

其论文发表期刊：1、*COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION* 在中科院文献情报中心期刊分区表(2021)的数学(大类）学科中为 1 区 TOP 期刊；2、*IET CONTROL THEORY AND APPLICATIONS* 在中科院文献情报中心期刊分区表（2021）的工程技术（大类）学科中为 3 区期刊。

检索结果详见附件（共计 3 页）。

特此证明！

检索证明人：龚凯丽　　审核人：周蓓蓓

教育部科技查新工作站（L28）
南京理工大学图书馆
2023 年 8 月 27 日

210094 南京市孝陵卫 200 号　Tel：025-84314962　Email:tsgxx@mail.njust.edu.cn

Web of Science™

2 record(s) printed from Clarivate Web of Science

第 1 条，共 2 条

标题: Networked sampled-data control of distributed parameter systems via distributed sensor networks

作者: Ji, HH (Ji, Huihui); Cui, BT (Cui, Baotong); Liu, XZ (Liu, Xinzhi)

来源出版物: COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION **卷:** 98 **文献号:** 105773 **DOI:** 10.1016/j.cnsns.2021.105773 **提前访问日期:** FEB 2021 **出版年:** JUL 2021

Web of Science 核心合集中的 "被引频次": 6

被引频次合计: 6

入藏号: WOS:000635186100001

文献类型: Article

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Math & Stat, 86 West Yushan Rd, Nanjing 211815, Peoples R China.
[Cui, Baotong] Jiangnan Univ, Minist Educ, Key Lab Adv Proc Control Light Ind, Wuxi 214122, Jiangsu, Peoples R China.
[Cui, Baotong] Jiangnan Univ, Sch IoT Engn, Wuxi 214122, Jiangsu, Peoples R China.
[Liu, Xinzhi] Univ Waterloo, Dept Appl Math, Waterloo, ON N2L 3G1, Canada.

通讯作者地址: Ji, HH (通讯作者)，Nanjing Audit Univ, Sch Math & Stat, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

电子邮件地址: jihuihui2020@nau.edu.cn; btcui@jiangnan.edu.cn; xzliu@uwaterloo.ca

第 2 条，共 2 条

标题: Event-triggered predictor-based control of distributed parameter systems

作者: Ji, HH (Ji, Huihui); Cui, BT (Cui, Baotong); Liu, XZ (Liu, XinZhi)

来源出版物: IET CONTROL THEORY AND APPLICATIONS **卷:** 15 **期:** 5 **页:** 721-736 **DOI:** 10.1049/cth2.12073 **提前访问日期:** DEC 2020 **出版年:** MAR 2021

Web of Science 核心合集中的 "被引频次": 1

被引频次合计: 1

入藏号: WOS:000604125800001

文献类型: Article

地址: [Ji, Huihui] Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.
[Liu, XinZhi] Univ Waterloo, Sch Appl Math, Waterloo, ON N2L 3G1, Canada.

通讯作者地址: Ji, HH (通讯作者)，Nanjing Audit Univ, Sch Stat & Math, 86 West Yushan Rd, Nanjing 211815, Peoples R China.

电子邮件地址: jihuihui2009@163.com

End of File

期刊全称：	Communications in Nonlinear Science and Numerical Simulation					
期刊简称：	COMMUN NONLINEAR SCI		ISSN：	1007-5704		
年份：	2021年		综述：	否		
	学科名称				分区	Top期刊
小类	MATHEMATICS, APPLIED应用数学				1	-
小类	MATHEMATICS, INTERDISCIPLINARY APPLICATIONS数学跨学科应用				1	-
小类	MECHANICS力学				2	-
小类	PHYSICS, FLUIDS & PLASMAS物理：流体与等离子体				1	-
小类	PHYSICS, MATHEMATICAL物理：数学物理				1	-
大类	数学				1	是

期刊影响因子				总被引频次		
2018年	2019年	2020年	2018-2020年平均	2019年	2020年	2019年-2020年
3.967	4.115	4.260	4.114	12329	14765	27094

备注：

说说您的看法（AddJComment.aspx?title=Communications+in+Nonlinear+Science+and+Numerical+Simulation&issn=1007-5704&year=2021&department=南京理工大学&ip=112.25.223.102）

期刊全称：	IET Control Theory and Applications					
期刊简称：	IET CONTROL THEORY A		ISSN：	1751-8644		
年份：	2021年		综述：	否		

	学科名称	分区	Top期刊
小类	AUTOMATION & CONTROL SYSTEMS自动化与控制系统	3	-
小类	ENGINEERING, ELECTRICAL & ELECTRONIC工程：电子与电气	3	-
小类	INSTRUMENTS & INSTRUMENTATION仪器仪表	2	-
大类	工程技术	3	否

期刊影响因子				总被引频次		
2018年	2019年	2020年	2018-2020年平均	2019年	2020年	2019年-2020年
3.526	3.343	3.527	3.465	8082	9676	17758

备注：

说说您的看法（AddJComment.aspx?title=IET+Control+Theory+and+Applications&issn=1751-8644&year=2021&department=南京理工大学&ip=112.25.223.106)

南京理工大学图书馆专用章

科研项目 高教项目 科研成果 学术活动 高被引 校级项目申报 纵向项目申报 教研项目申报 入账办理 更多 季慧慧

成果列表

考核时间更新 新增 待确认网推论文

名称 模糊 审核状态 所属单位 第一作者姓名 模糊 更多 查询

名称	所有作者	成果日期	成果类型	审核状态	操作
Neural-Network-Based Stochastic Scheduling Control of Unknown Nonlinear Systems	季慧慧,徐胜元（外）,张贺（外）	2024-01-01	论文	学校通过	收录补录 转载补录
Networked sampled-data control of distributed parameter systems via distributed sensor networks SCIE	季慧慧,Cui, Baotong（外）,Liu, Xinzhi（外）	2021-07-01	论文	学校通过	收录补录 转载补录
Event-triggered predictor-based control of distributed parameter systems	季慧慧,崔宝同（外）,刘新芝（外）	2021-03-01	论文	学校通过	收录补录 转载补录
2023年度江苏省自动化学会科学技术奖	张贺（外）,喻果（外）,黄书华（外）,季慧慧,彭岳（外）,蒲翠梅（外）	2024-02-23	获奖	学校通过	
Finite-time quasi- synchronization of stochastic multilayer networks with energy consumption estimation	徐玉华,Wu Xiaoqun（外）,Wan Xiaoxiao（外）,季慧慧,周世超	2022-04-16	论文	学校通过	
Event-generator-based H∞ control of fuzzy distributed parameter systems	季慧慧,崔宝同（外）	2022-03-28	论文	学校通过	收录补录 转载补录
概率论与数理统计课程思政的探索与思考 知	季慧慧	2022-01-05	论文	学校通过	收录补录 转载补录

成果查看

基本信息　收录转载　依托项目　审核记录　修改记录

基本信息

名称	Event-triggered predictor-based control of distributed parameter systems		
发表/出版时间	2021-03-01	发表/刊物论文集	IET Control Theory and Applications
教育部统计归属	科技类	期刊来源类型	外文期刊
论文类型	期刊论文	所属单位	统计与数据科学学院
刊物级别	SCI 二区		
SCI论文分区		SSCI分区	
影响因子		项目来源	其他课题
一级学科	工程与技术科学基础学科	奖励级别	重点奖励一档
字数	1.1 万字	刊型	正刊
是否符合审计提升	否	提升后级别	重点奖励一档
文献类型			

详细信息

发表范围	国外学术期刊	学校署名	第一单位
卷期页		DOI	
ISSN号		CN号	
附件	IET2020.pdf 下载 预览		

关闭

5. 主持国家自然科学基金青年项目 1 项(1/1)

国家自然科学基金资助项目批准通知

（包干制项目）

季慧慧　先生/女士：

根据《国家自然科学基金条例》、相关项目管理办法规定和专家评审意见，国家自然科学基金委员会（以下简称自然科学基金委）决定资助您申请的项目。项目批准号：62403247，项目名称：拒绝服务攻击下实时随机多任务调度系统网络容侵控制研究，资助经费：30.00万元，项目起止年月：2025年01月至 2027年 12月，有关项目的评审意见及修改意见附后。

请您尽快登录科学基金网络信息系统（https://grants.nsfc.gov.cn），**认真阅读《国家自然科学基金资助项目计划书填报说明》并按要求填写《国家自然科学基金资助项目计划书》（以下简称计划书）**。对于有修改意见的项目，请您按修改意见及时调整计划书相关内容；如您对修改意见有异议，须在电子版计划书报送截止日期前向相关科学处提出。

请您将电子版计划书通过科学基金网络信息系统（https://grants.nsfc.gov.cn）提交，由依托单位审核后提交至自然科学基金委。自然科学基金委审核未通过者，将退回的电子版计划书修改后再行提交；审核通过者，打印纸质版计划书（一式两份，双面打印）并在项目负责人承诺栏签字，由依托单位在承诺栏加盖依托单位公章，且将申请书纸质签字盖章页订在其中一份计划书之后，一并报送至自然科学基金委项目材料接收工作组。纸质版计划书应当保证与审核通过的电子版计划书内容一致。**自然科学基金委将对申请书纸质签字盖章页进行审核，对存在问题的，允许依托单位进行一次修改或补齐。**

向自然科学基金委提交电子版计划书、报送纸质版计划书并补交申请书纸质签字盖章页截止时间节点如下：

1. **2024年9月9日16点：**提交电子版计划书的截止时间；
2. **2024年9月16日16点：**提交修改后电子版计划书的截止时间；
3. **2024年9月23日：**报送纸质版计划书（一式两份，其中一份包含申请书纸质签字盖章页）的截止时间。
4. **2024年10月8日：**报送修改后的申请书纸质签字盖章页的截止时间。

请按照以上规定及时提交电子版计划书，并报送纸质版计划书和申请书纸质签字盖章页，逾期不报计划书或申请书纸质签字盖章页且未说明理由的，视为自动放弃接受资助；未按要求修改或逾期提交申请书纸质签字盖章页者，将视情况给予暂缓拨付经费等处理。

附件：项目评审意见及修改意见表

国家自然科学基金委员会

2024年8月23日

6. 获 2024 年度江苏省自动化学会科学技术三等奖(1/5)

证书

2024年度江苏省自动化学会科学技术奖

为表彰江苏省自动化学会科学技术奖获得者，特颁发此证书。

项目名称：工业复杂系统随机控制理论与关键技术

奖励等级：三等

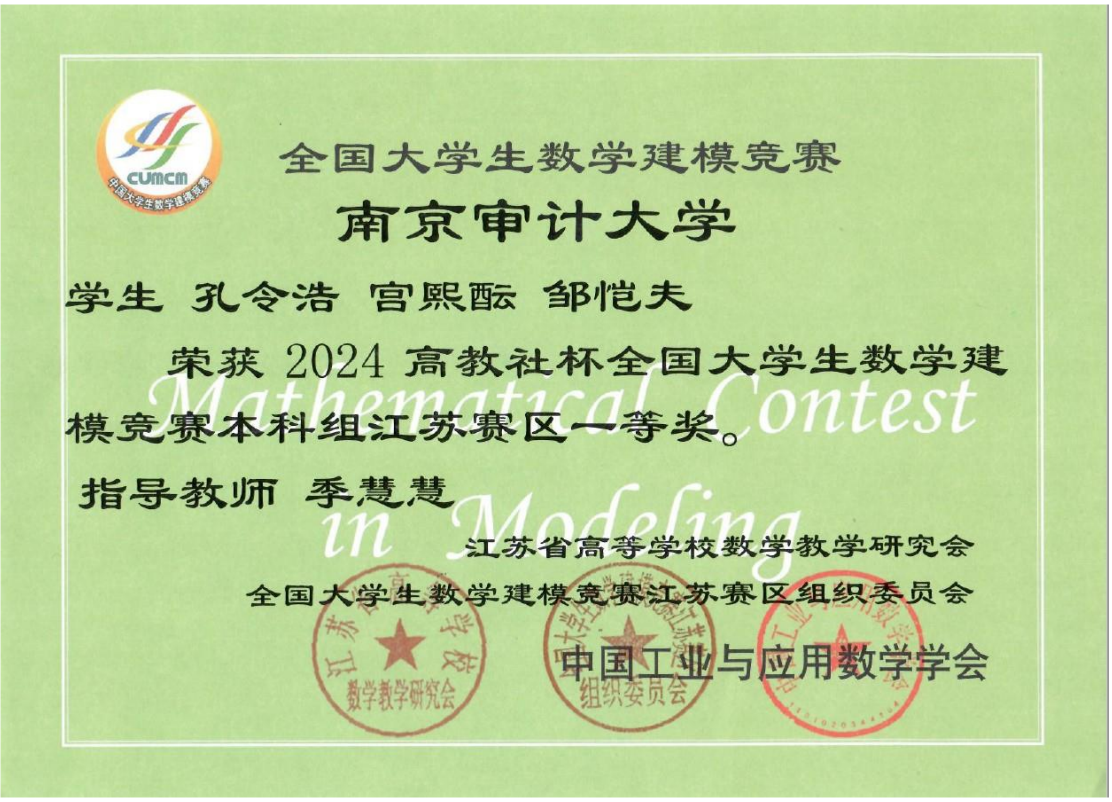
获 奖 者：李慧慧

江苏省自动化学会

2025年1月27日

证书号：2024-3-06-R1

7. 指导学生获高教社杯全国大学生数学建模竞赛本科组江苏赛区一等奖 (1/1)



全国大学生数学建模竞赛

南京审计大学

学生 孔令浩 宫熙酝 邹恺夫

荣获 2024 高教社杯全国大学生数学建模竞赛本科组江苏赛区一等奖。

指导教师 季慧慧

江苏省高等学校数学教学研究会

全国大学生数学建模竞赛江苏赛区组织委员会

中国工业与应用数学学会